Gravitational Wave Astrophysics

Lecture 2

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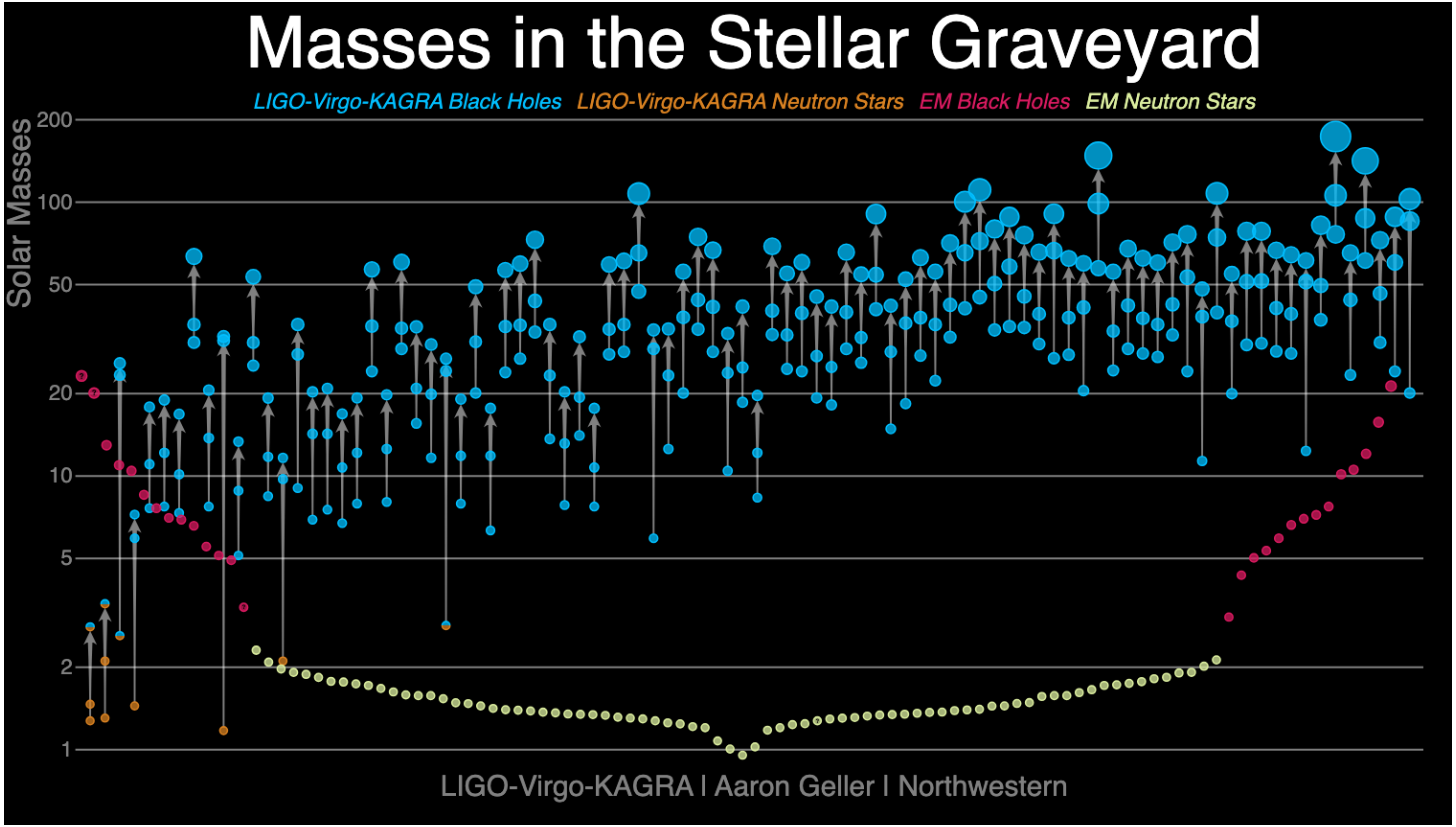
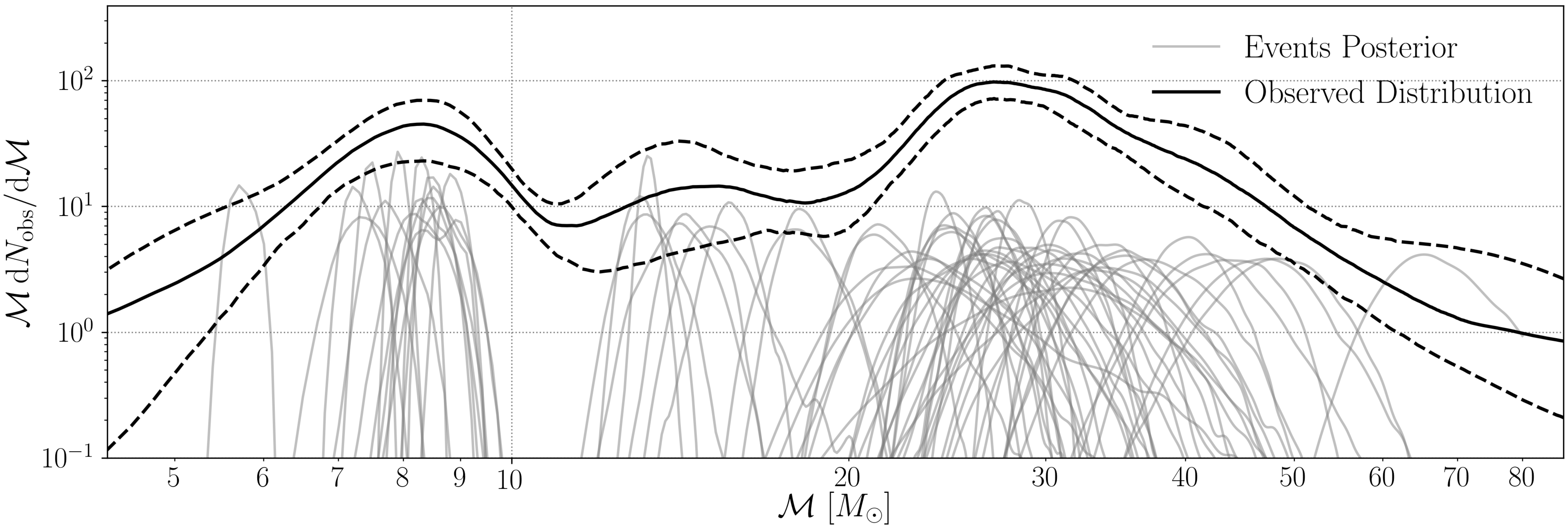
In this lecture, you'll learn

- Population analysis of GW events
- Astrophysics of compact objects:
    - Isolated formation channel
    - Dynamical formation channel

Population studies



Population studies



Credits: The LIGO-Virgo-KAGRA collaboration 2021

Parametric models

Hierarchical Bayesian Analysis

$$p(\theta | d) \propto \mathscr{L}(d | \theta) p(\theta)$$

$$\theta = \{m_1, m_2, d_L, \ldots\}$$

$$\Big\downarrow$$

$$p(\Lambda | \{d\}) \propto \mathscr{L}(\{d\} | \Lambda) p(\Lambda)$$

$$\{d\} = \{d_1, d_2, d_3, \ldots\}$$

Parametric models

$$p(\Lambda \mid \{d\}) \propto \mathscr{L}(\{d\} \mid \Lambda) p(\Lambda)$$

Credits: Mandel et al. 2019, Abbott et al. 2020, Vitale et al. 2022

Parametric models

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$$\mathscr{L}(\{d\} \mid \Lambda) \propto \prod_{i}^{N_{obs}} \frac{\int \mathscr{L}(d_i \mid \theta) \pi(\theta \mid \Lambda)}{\xi(\Lambda)}$$

Credits: Mandel et al. 2019, Abbott et al. 2020, Vitale et al. 2022

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Credits: Mandel et al. 2019, Abbott et al. 2020, Vitale et al. 2022

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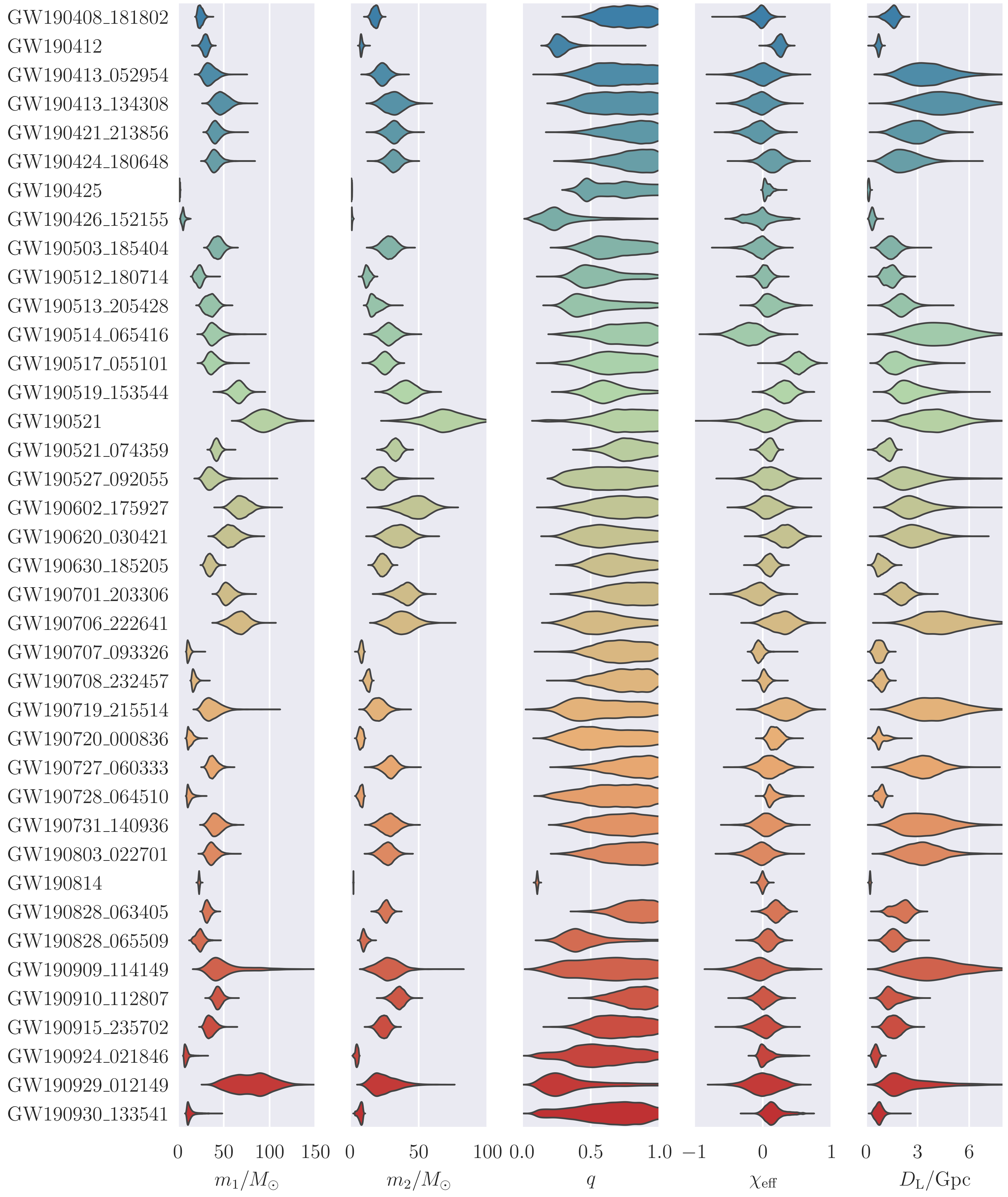


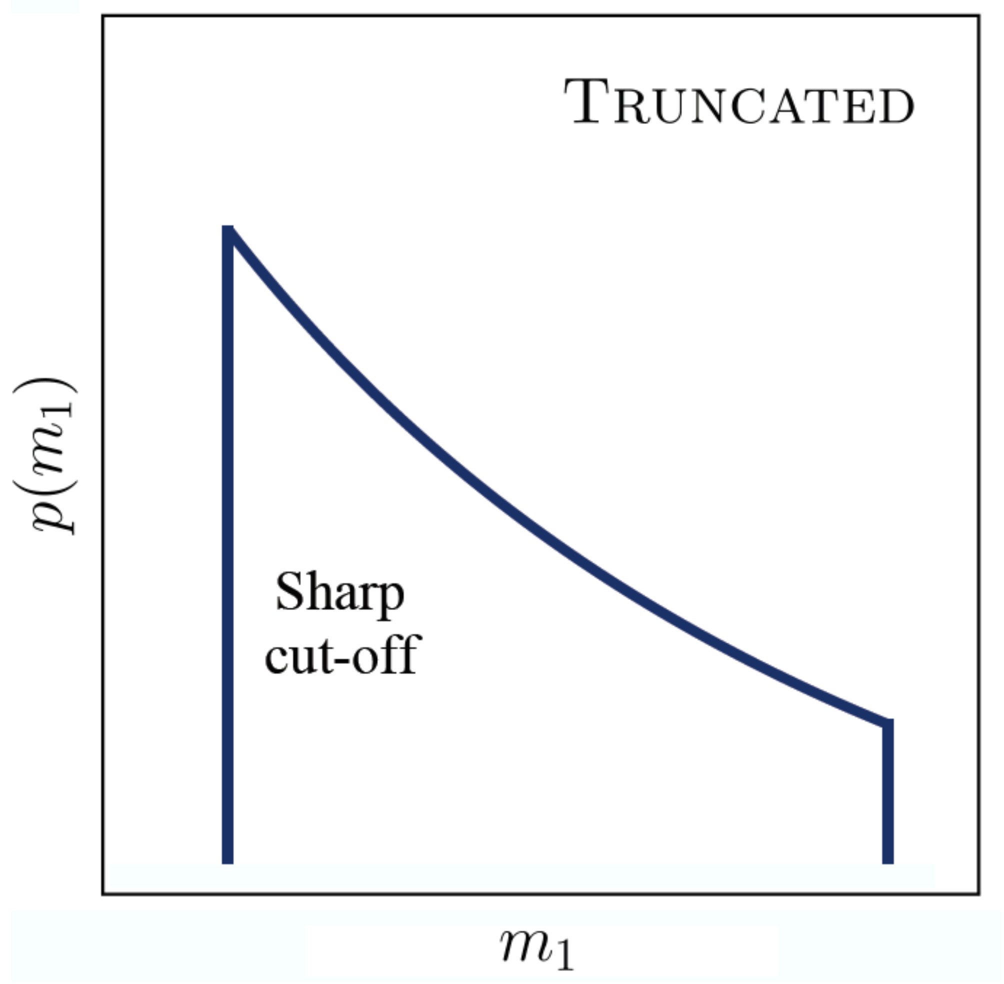
Credits: Abbott et al. 2020

Parametric models

$$p(\Lambda \,|\, \{d\}) \propto \mathscr{L}(\{d\} \,|\, \Lambda) p(\Lambda)$$

Population models

$$\mathscr{L}(\{d\} \,|\, \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathscr{L}(d_i \,|\, \theta) \pi(\theta \,|\, \Lambda)}{\xi(\Lambda)}$$



Why a power law?

$$\pi(m_1 | \alpha, m_{\min}, m_{\max}) \propto \begin{cases} m_1^{-\alpha} & m_{\min} < m_1 < m_{\max} \\ 0 & \text{otherwise,} \end{cases}$$

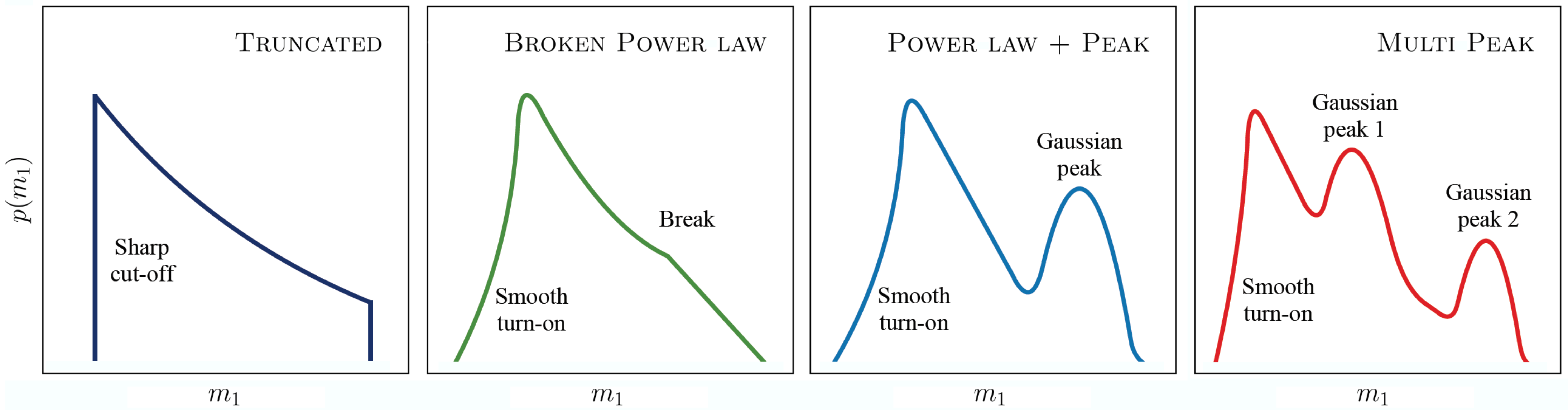
Credits: Abbott et al. 2021

Parametric models

$$p(\Lambda \,|\, \{d\}) \propto \mathscr{L}(\{d\} \,|\, \Lambda) p(\Lambda)$$

Population models

$$\mathscr{L}(\{d\} \,|\, \Lambda) \propto \prod_i^{N_{obs}} \frac{\int \mathscr{L}(d_i \,|\, \theta) \pi(\theta \,|\, \Lambda)}{\xi(\Lambda)}$$



Credits: Abbott et al. 2021

Parametric models

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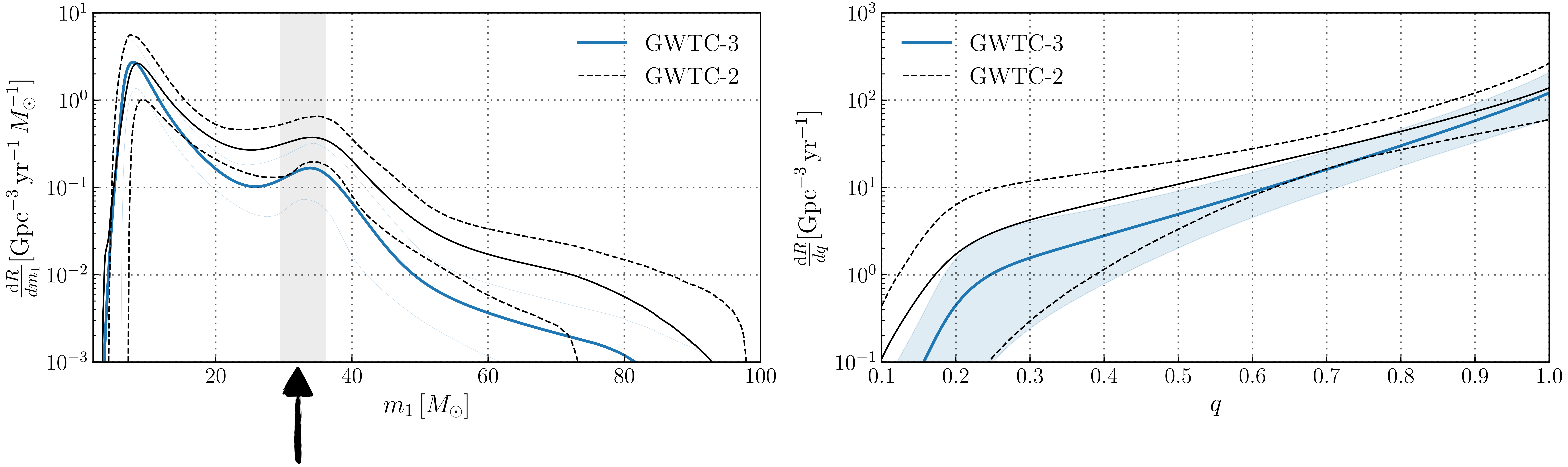
Selection effects: fraction of merger that are detectable for a population with parameters Λ
We will define it in lecture #4

Masses

$$p(\Lambda \,|\, \{d\}) \propto \mathscr{L}(\{d\} \,|\, \Lambda) p(\Lambda)$$

Power Law + Peak



Mean of the Gaussian peak

The LIGO-Virgo-KAGRA collaboration 2021

Spins

- $\chi_{\mathrm{eff}} = (\mathbf{S}_1/m_1 + \mathbf{S}_2/m_2)\cdot\hat{\mathbf{L}}/M = \dfrac{\chi_1 \cos\theta_1 + q\chi_2 \cos\theta_2}{1+q}$

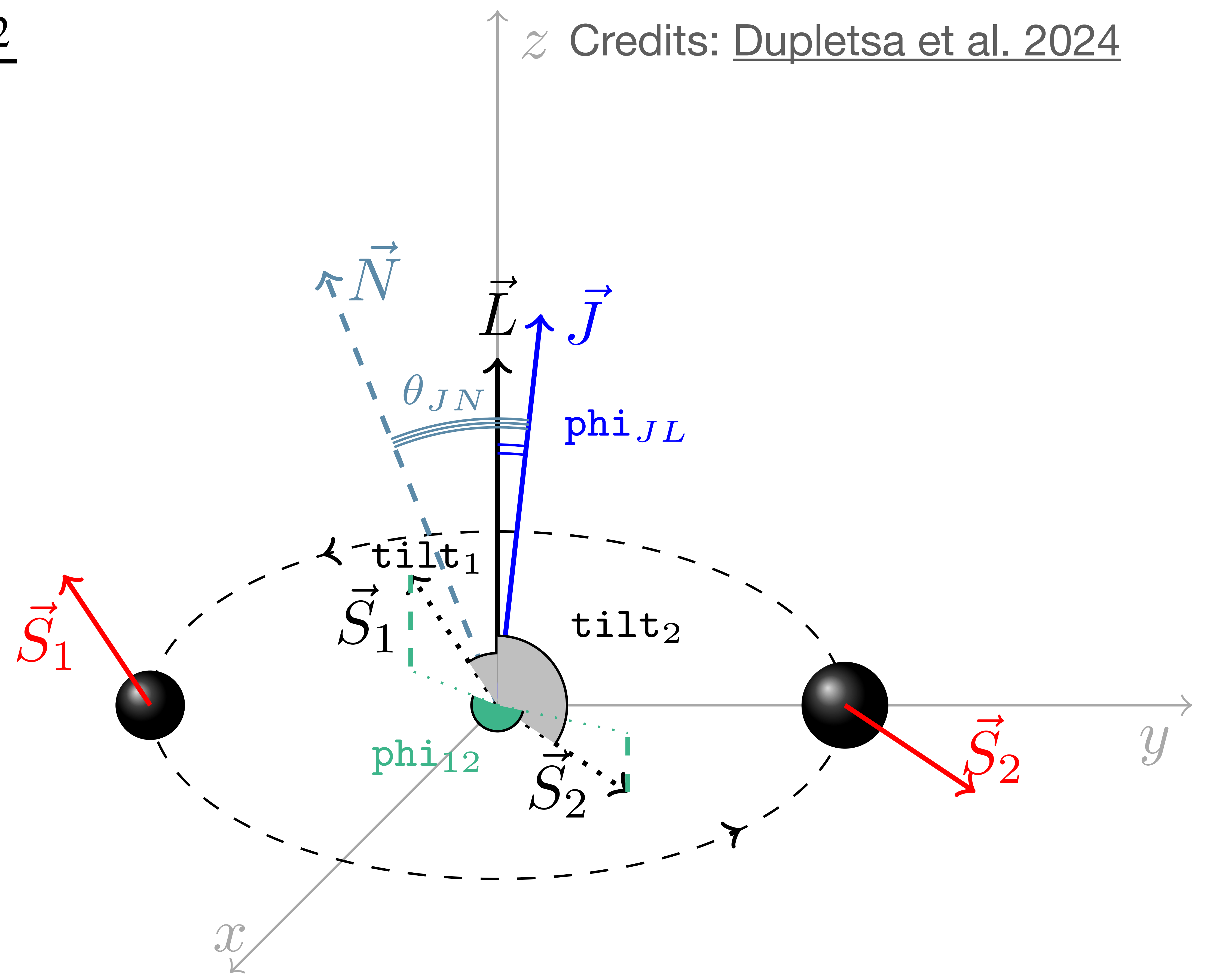
- $\chi_p = \max\left[\chi_1 \sin\theta_1, \left(\dfrac{3+4q}{4+3q}\right) q\chi_2 \sin\theta_2\right]$

- χ_{eff} and χ_p are approximately conserved quantities

- Dimension-less spin component $\chi_i = \mathbf{S}_i/m_i$ where $\mathbf{S}_i$ is the individual spin

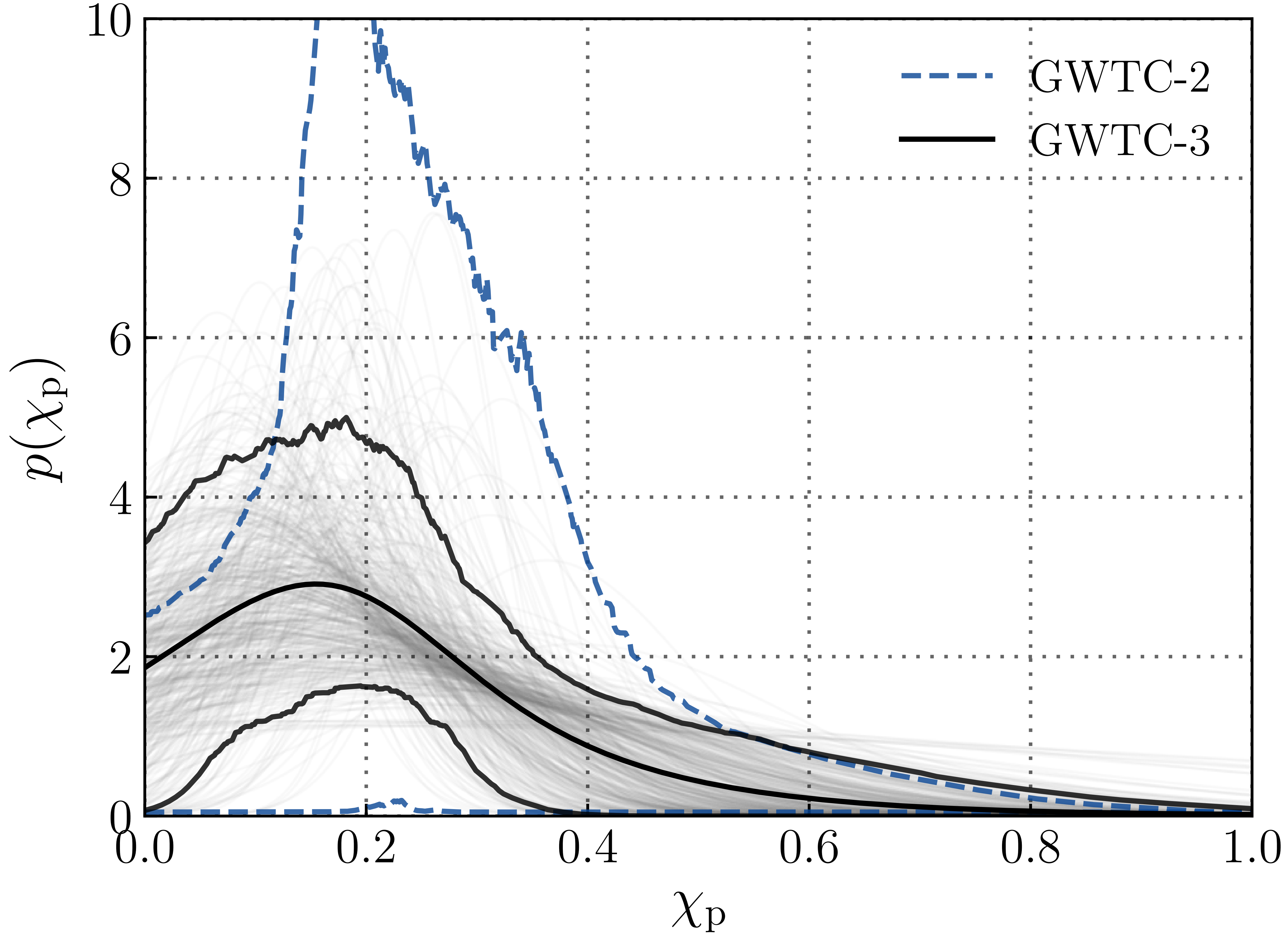
- $\hat{\mathbf{L}}$ is orbital angular momentum

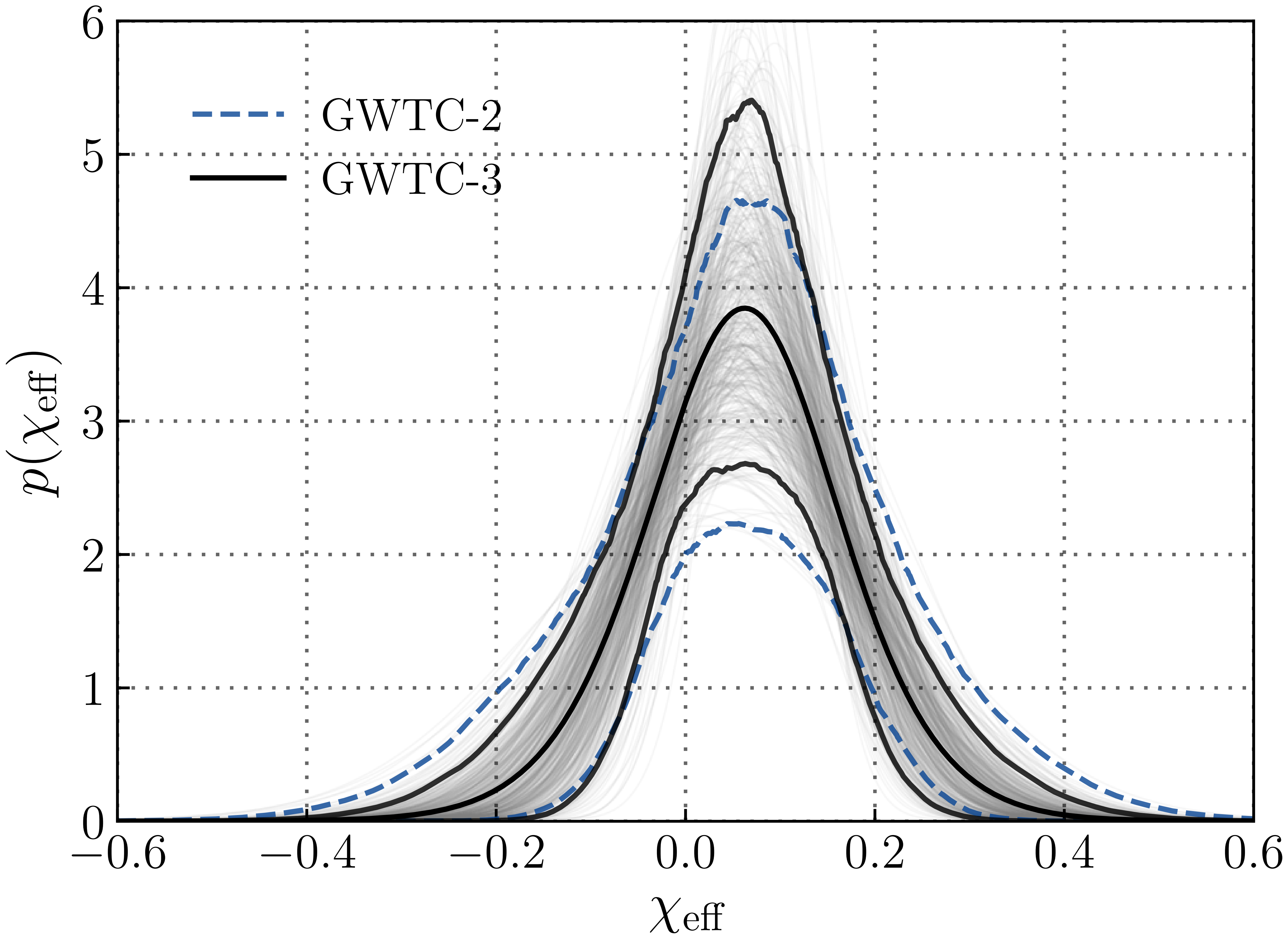
Credits: Dupletsa et al. 2024



Refs: Hannam et al. 2014, Schmidt et al. 2015

Spins



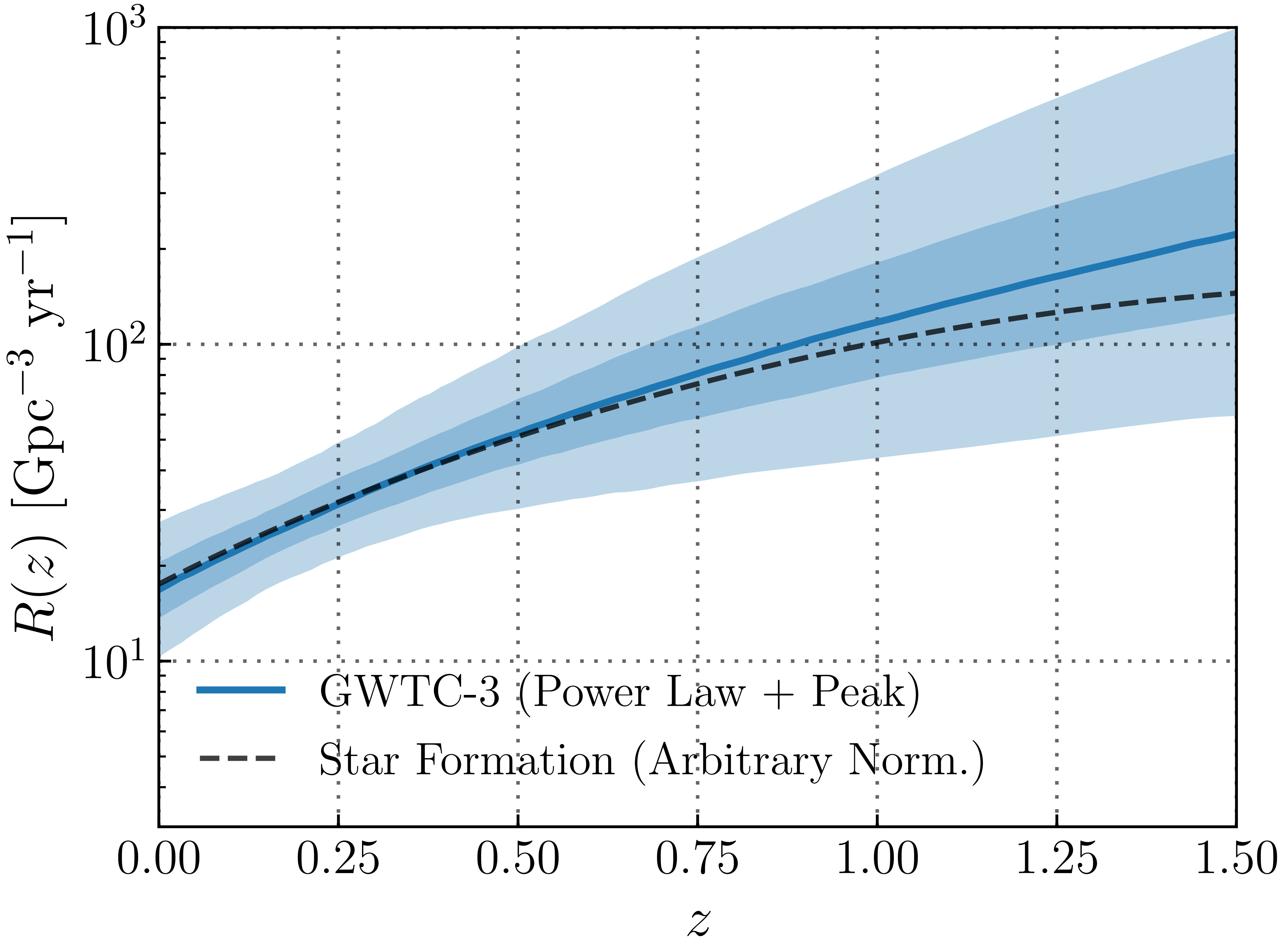


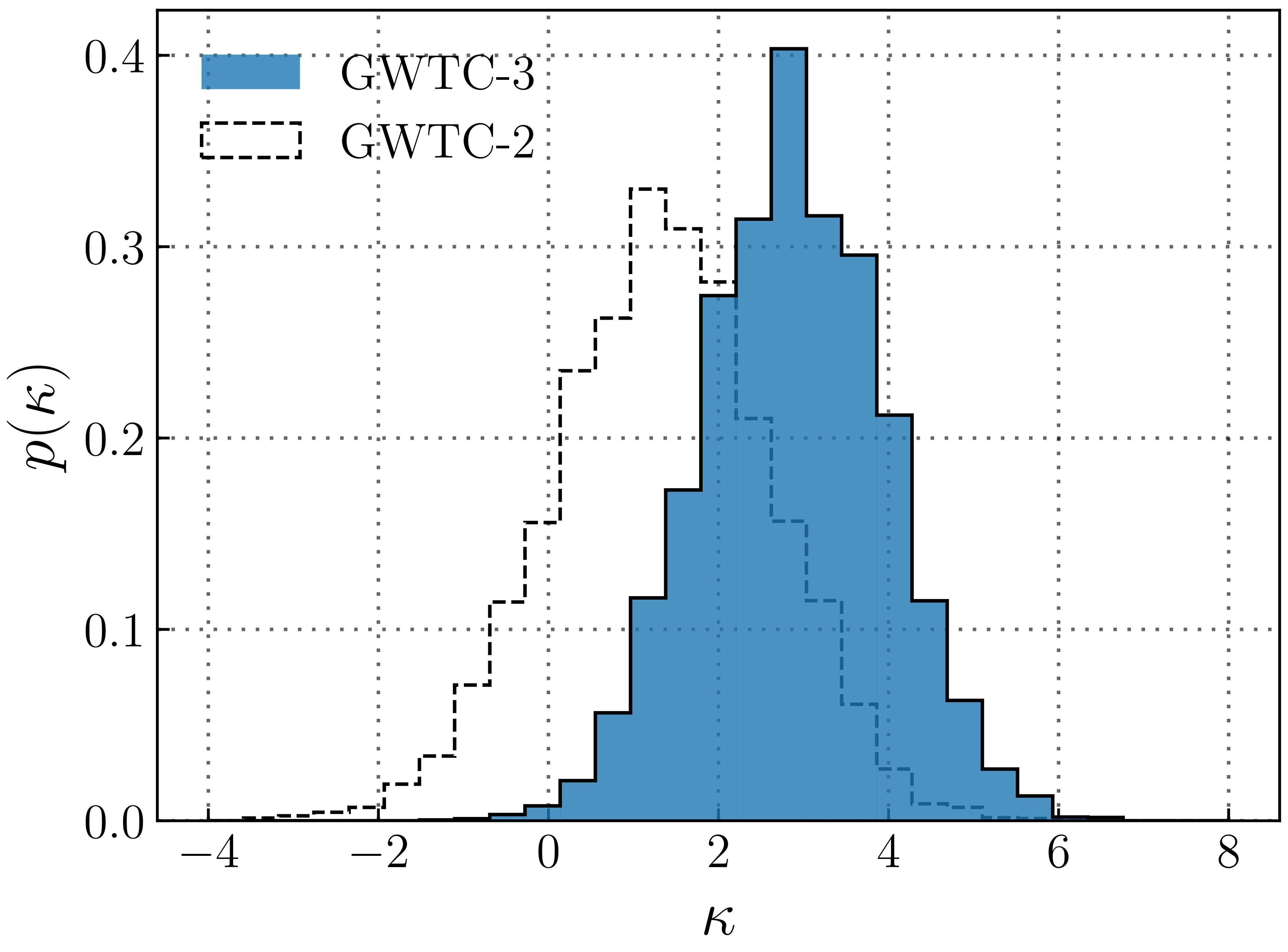
small but non-vanishing spins

The LIGO-Virgo-KAGRA collaboration 2021

Rates

$$\mathscr{R} \propto (1+z)^{\kappa}$$



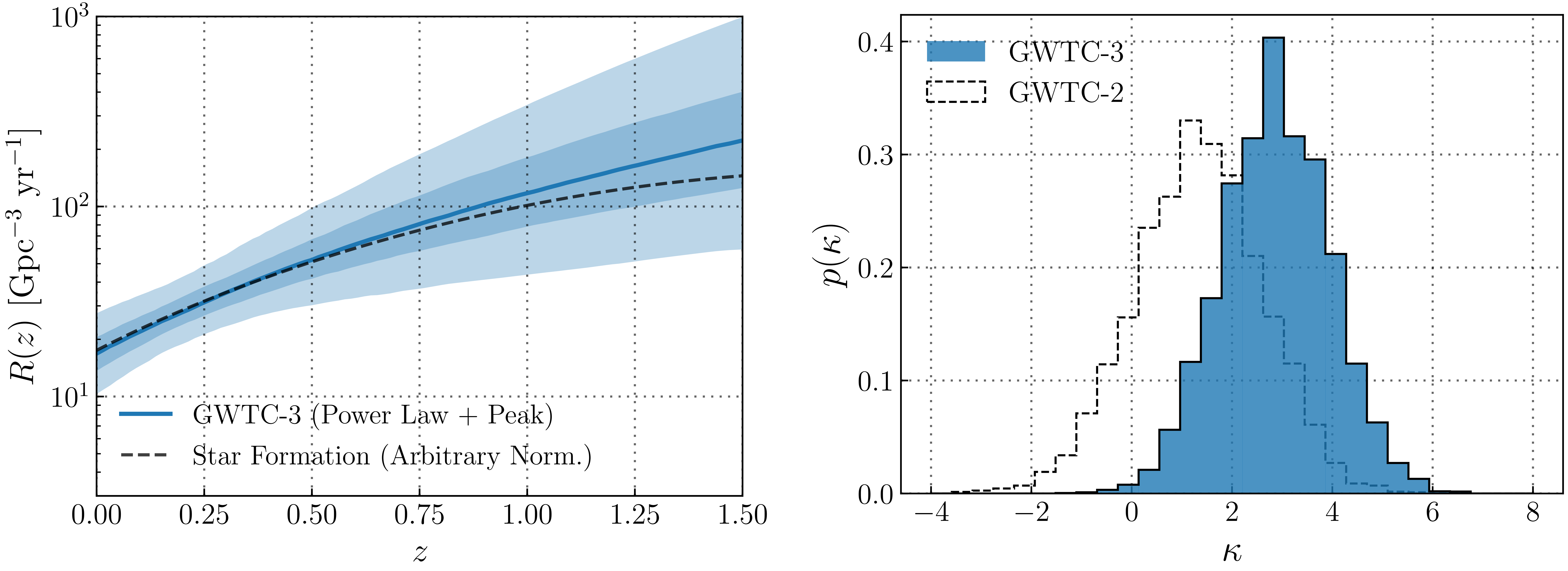


Merger rate density is increasing with redshift

Refs: The LIGO-Virgo-KAGRA collaboration 2021

Rates

$$\mathscr{R} \propto (1+z)^{\kappa}$$



Today's hands-on section!

Refs: The LIGO-Virgo-KAGRA collaboration 2021

Astrophysics of compact objects: can our models predict observations?

NS vs BH

- Final stage of massive star evolution
- **NS mass** $\in [\sim 1, \sim 3]\ \mathrm{M}_{\odot}$
    - Lower limit: **Chandrasekhar mass:** maximum mass for a star to be supported by electron degeneracy
    - Upper limit: **Tolman-Oppenheimer-Volkoff limit:** max mass above which no source pressure can counteract gravity
- **BH mass** $\gtrsim 3\ \mathrm{M}_{\odot}$

Refs: Chandrasekhar 1931, Oppenheimer and Volkoff 1939

GW decay

- $\frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} = -\frac{Gm_1m_2}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t}, \frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} \sim \frac{32}{5}\frac{G^4}{c^5}\frac{m_1m_2(m_1+m_2)}{a^3}$

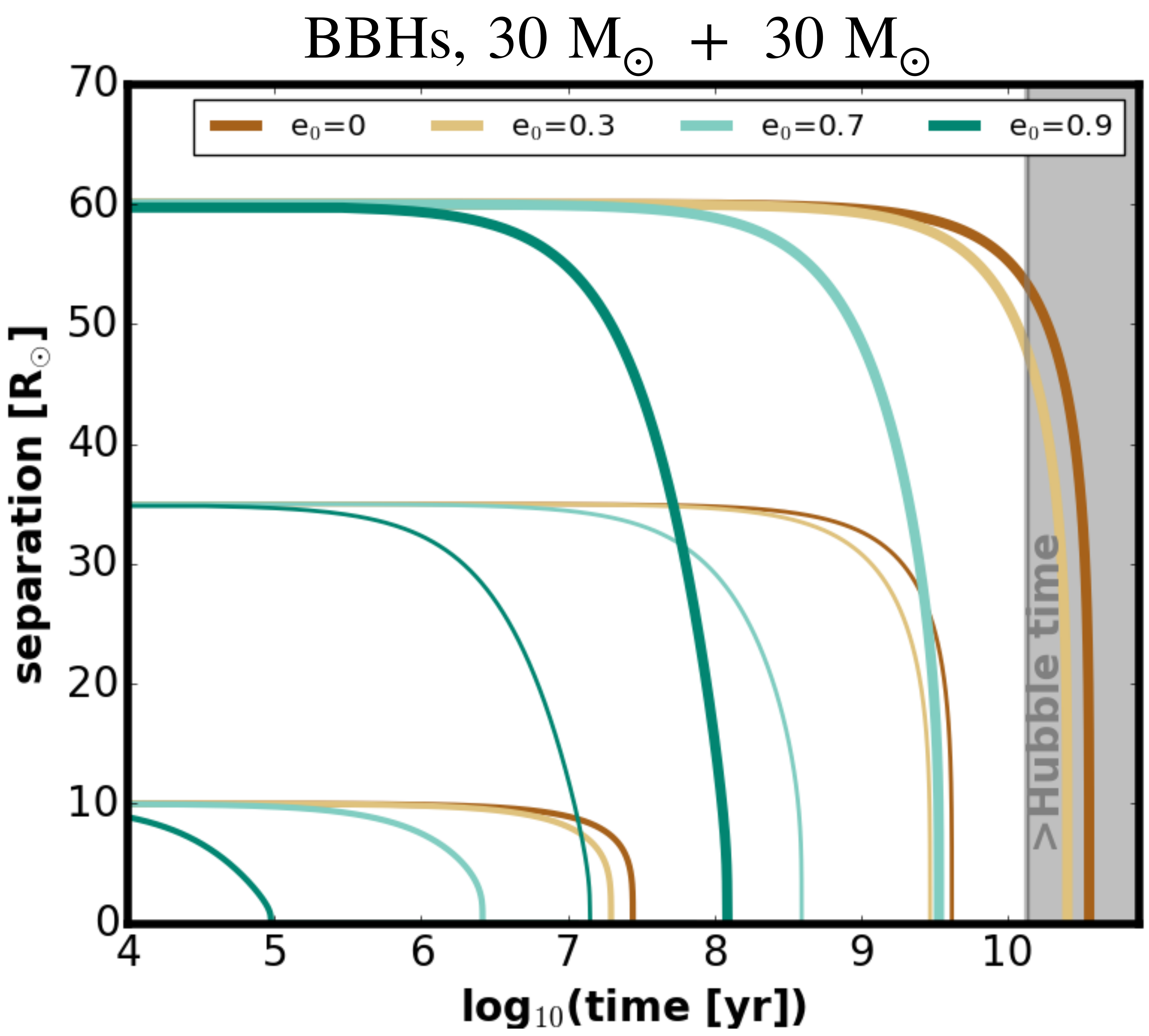
Refs: Peters 1964, Mapelli 2018, Iorio et al. 2023

GW decay

- $$\frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} = -\frac{Gm_1m_2}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t}, \quad \frac{\mathrm{d}E_{\mathrm{orb}}}{\mathrm{d}t} \sim \frac{32}{5}\frac{G^4}{c^5}\frac{m_1m_2(m_1+m_2)}{a^3}$$
- Orbital decay: $$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3m_1m_2(m_1+m_2)}{c^5a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$
- Circularisation: $$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{304}{15}e\frac{G^3m_1m_2(m_1+m_2)}{c^5a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right)$$

Refs: Peters 1964, Mapelli 2018, Iorio et al. 2023

GW decay



Credits: Martyna Chruślinska

Refs: Costa et al. 2023

GW decay

- Integrating $\frac{\mathrm{d}a}{\mathrm{d}t}$ and assuming $\frac{\mathrm{d}e}{\mathrm{d}t} = 0$:

$$t_{\rm GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1 m_2 (m_1 + m_2)}(1 - e^2)^{7/2}$$

- Example: $m_1 = m_2 = 1\ \mathrm{M}_\odot, a = 1\ \mathrm{AU}$ ➡️ $t_{\rm GW} \sim 2 \times 10^{17}\ \mathrm{yr}$

- GW decay is significant only in very tight binaries ➡️
  we need **astrophysical processes** to make the two compact objects merge

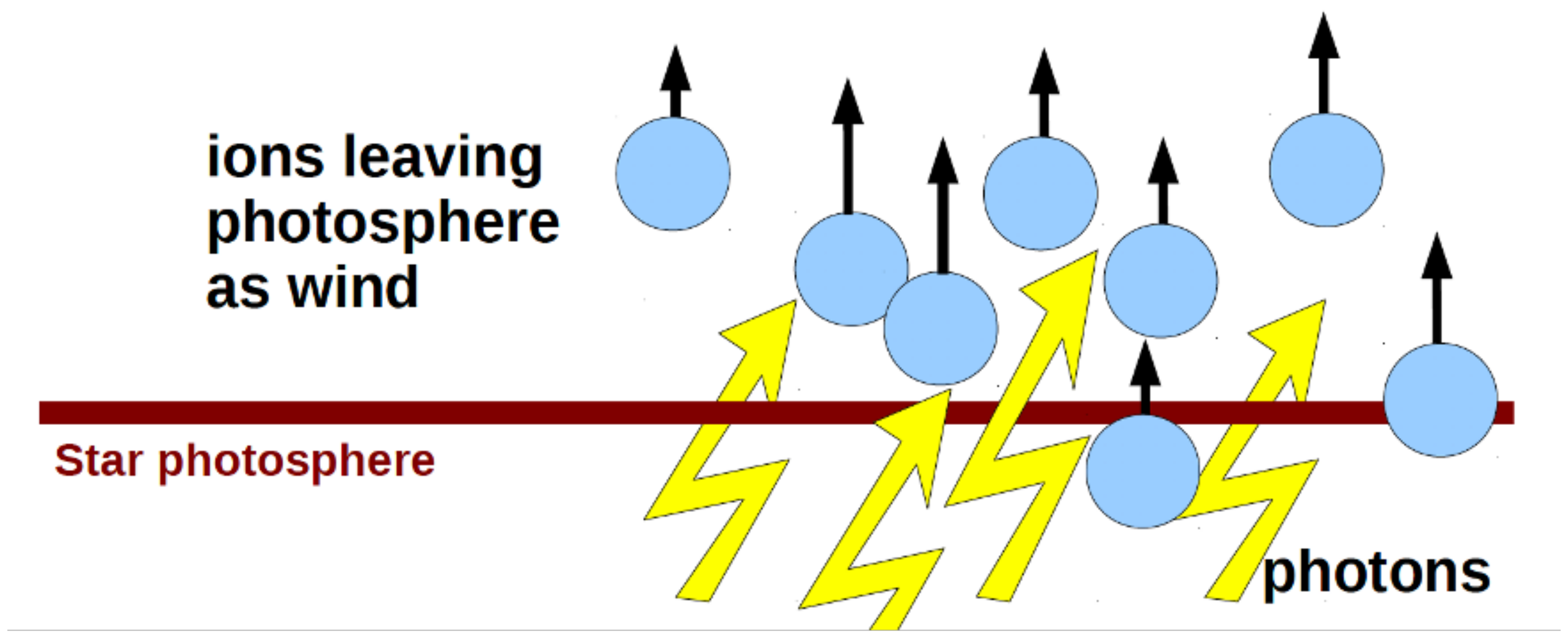
Refs: Peters 1964, Mapelli 2018, Iorio et al. 2023

Single stellar evolution

- **Metallicity:** fraction of every element heavier than hydrogen (X) and helium (Y) ➡️ $Z = 1 - X - Y$

- **Sun:** $Z_{\odot} \sim 0.015 - 0.02$ ➡️ metal rich star

- **Stellar winds:** photons in stellar atmosphere couples with ions ➡️ transfer of linear momentum that can unbind ions

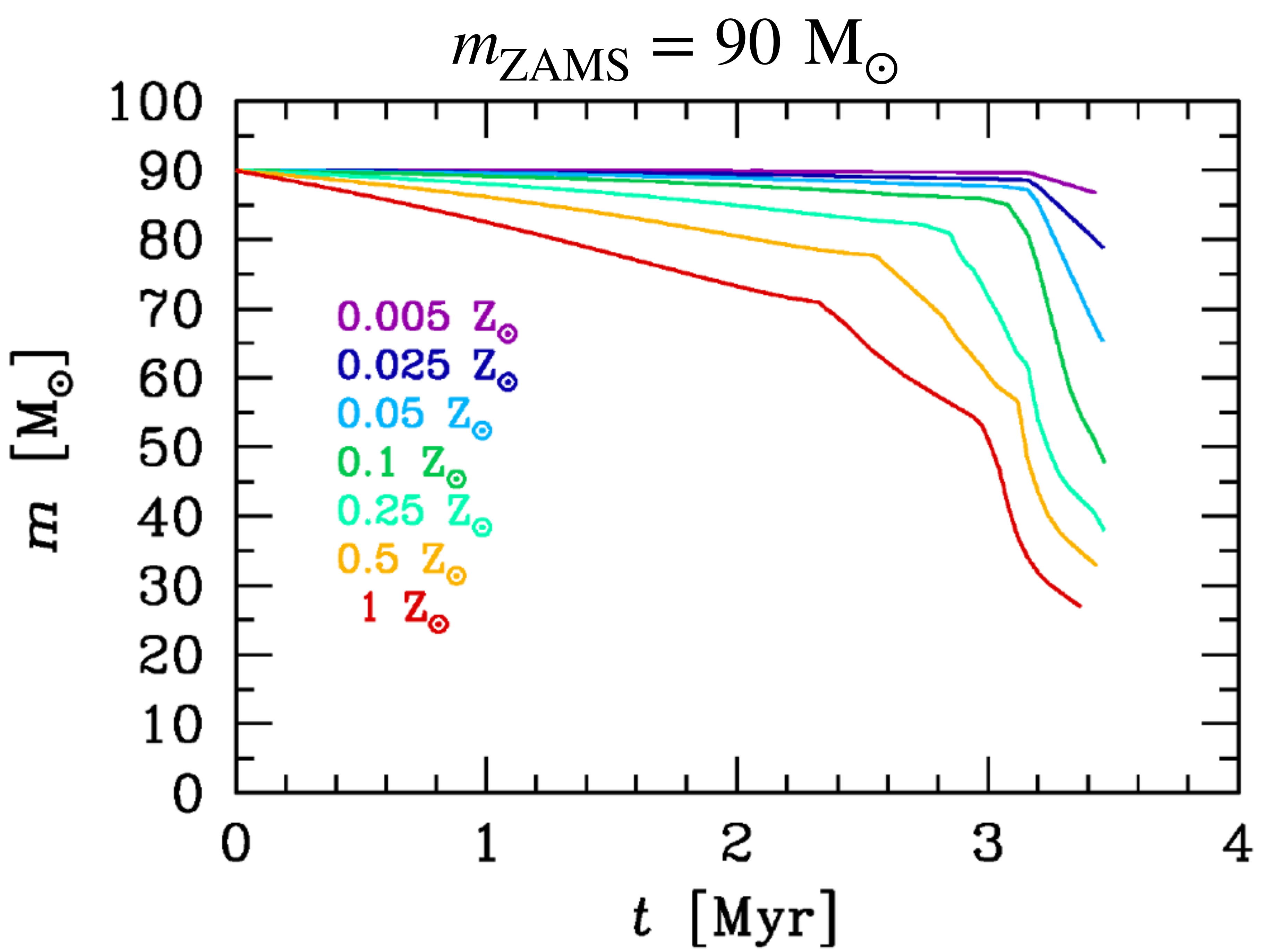


Credit: Michela Mapelli

Single stellar evolution

- Mass loss due to stellar winds depends on metallicity ➡️ $\dot{M} \propto Z^{\alpha}$ with $\alpha \sim 0.5 - 0.9$

- Massive and metal rich star can lose > 50% of their mass due to stellar winds



Credits: Michela Mapelli

Refs: Bressan et al. 2012, Spera et al. 2015, Vink et al. 2016, Mapelli 2018

Death of a star

- When nuclear burning is over, the star is out of hydrostatic equilibrium and the **core collapses** (CC)

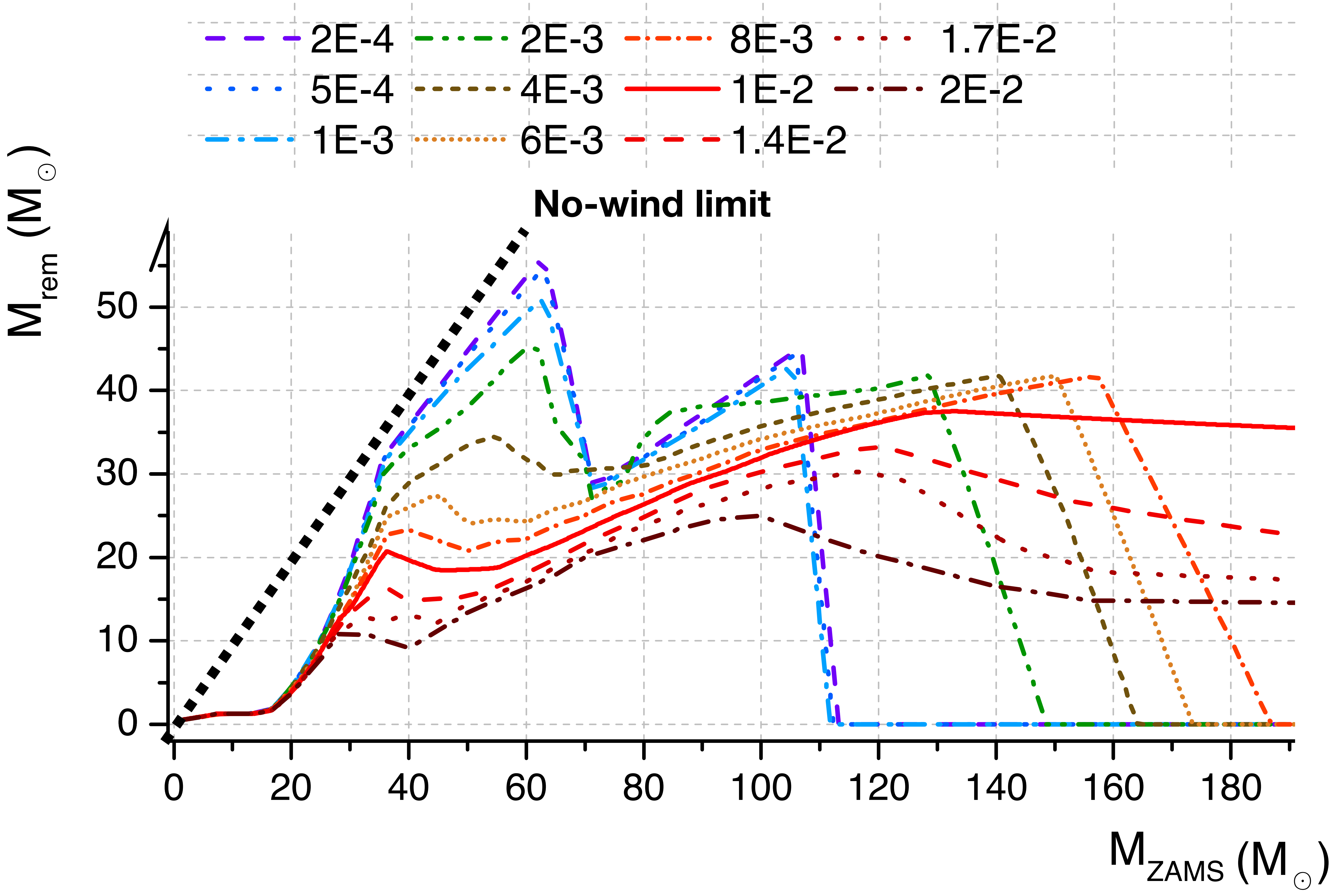
- $M_{\mathrm{ZAMS}} \lesssim 8\ \mathrm{M}_\odot$ ➡️ CC stops when electron degeneracy pressure balance gravity. Thermal pulses remove all the envelope, revealing the bare CO core ➡️ **White Dwarf**

- $M_{\mathrm{ZAMS}} \in [\sim 8, \sim 20]\ \mathrm{M}_\odot$ ➡️ CC stops when core reaches nuclear density and neutron degeneracy pressure balance gravity ➡️ **Neutron Star**

- $M_{\mathrm{ZAMS}} \gtrsim 20\ \mathrm{M}_\odot$ ➡️ gravity is too strong and no source of pressure can stop CC ➡️ **Black Hole**

Refs: Percival 2016, Couch et al. 2017

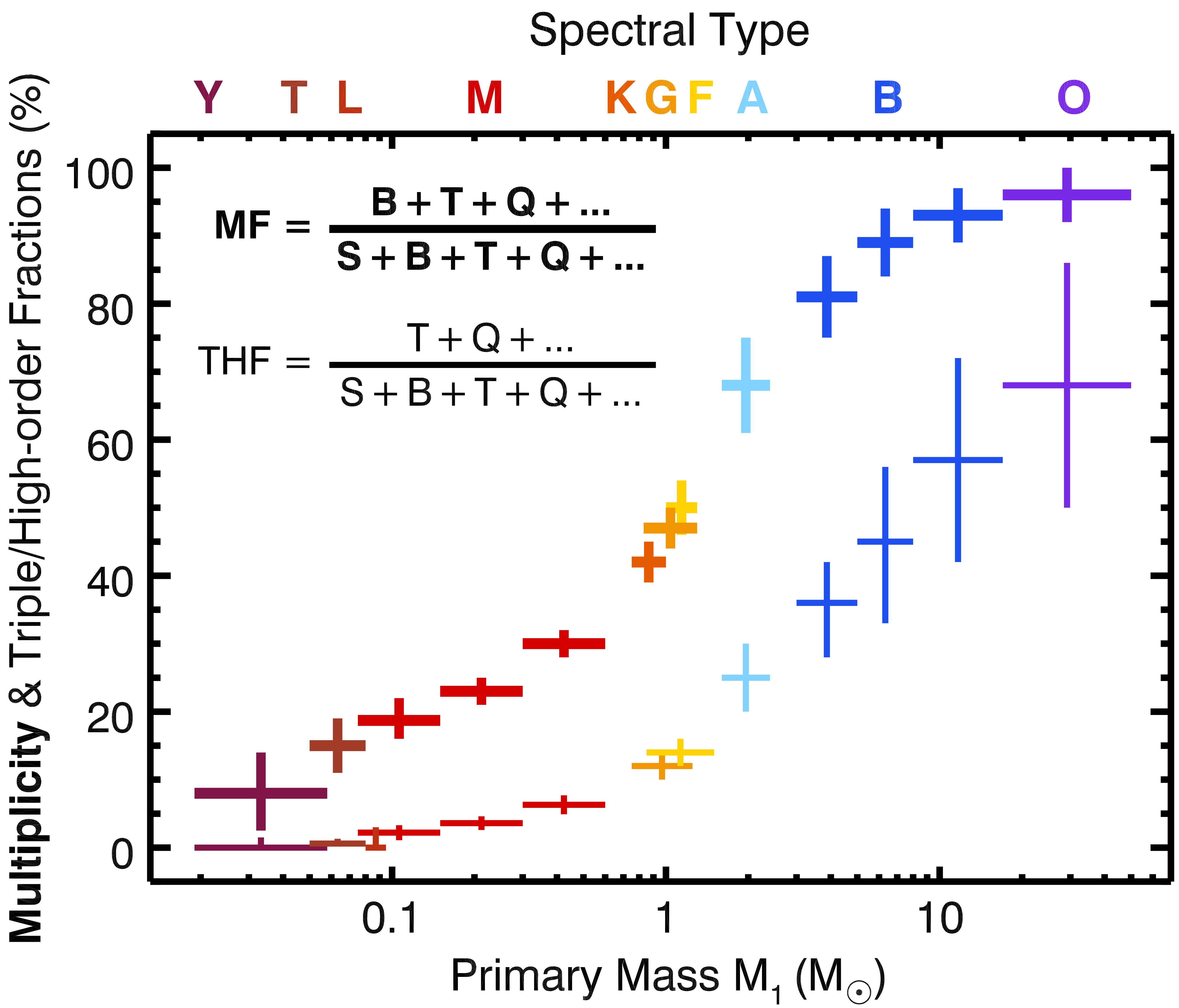
Single stellar evolution



Stellar winds set the upper limit of BH mass

Refs: Spera et al. 2015, Spera and Mapelli 2017, Mapelli 2018, Spera et al. 2022

Binary systems



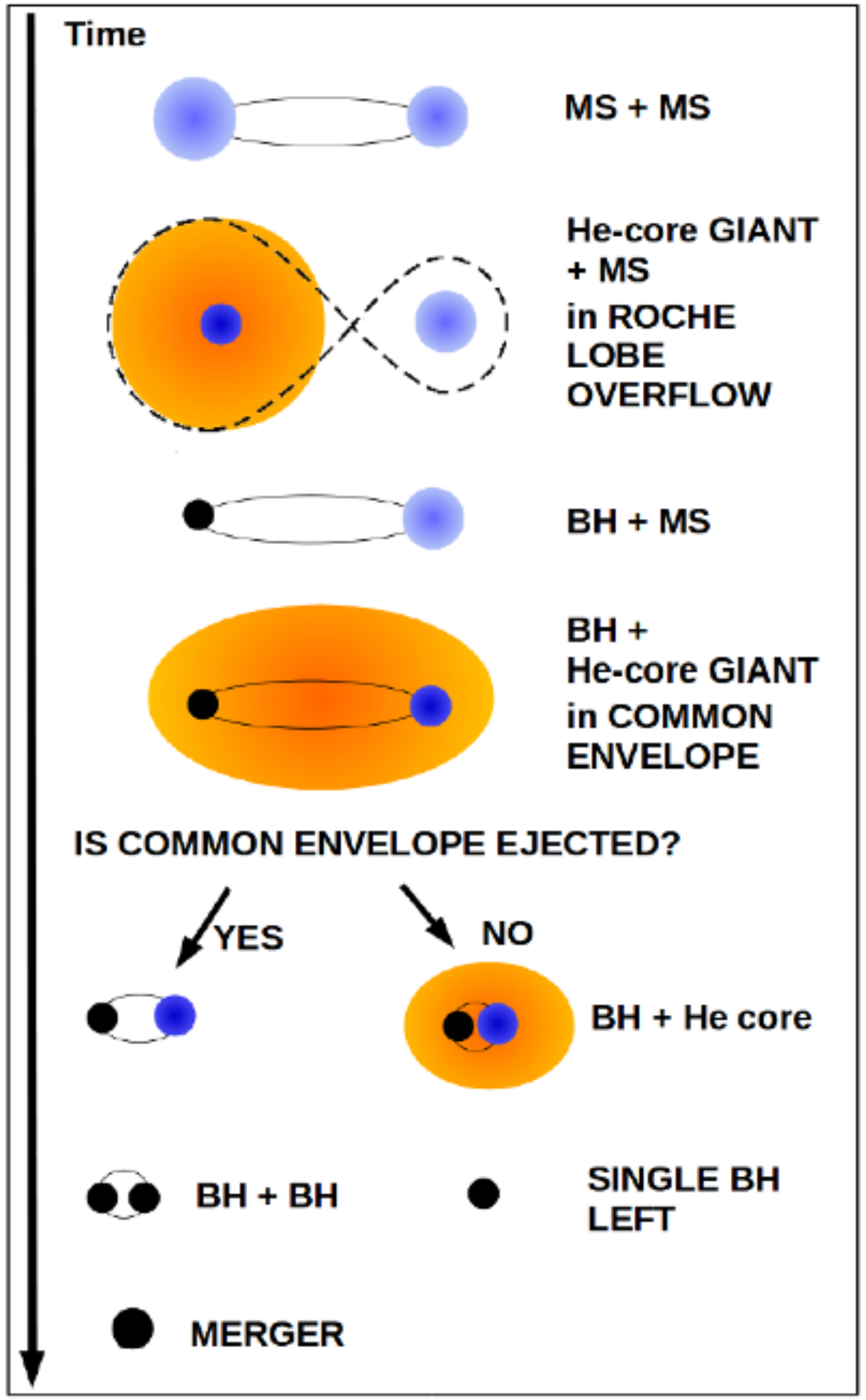
Most massive stars (> 70%) are in binary systems

Credits: Offner et al. 2023

Refs: Sana et al. 2012, Moe and Di Stefano 2017

Isolated formation channel

- **Two stars** form from the same gas cloud and evolve into **two merging BHs**
- Binary evolution driven by two main processes:
  - **Mass transfer**
  - **Common Envelope**



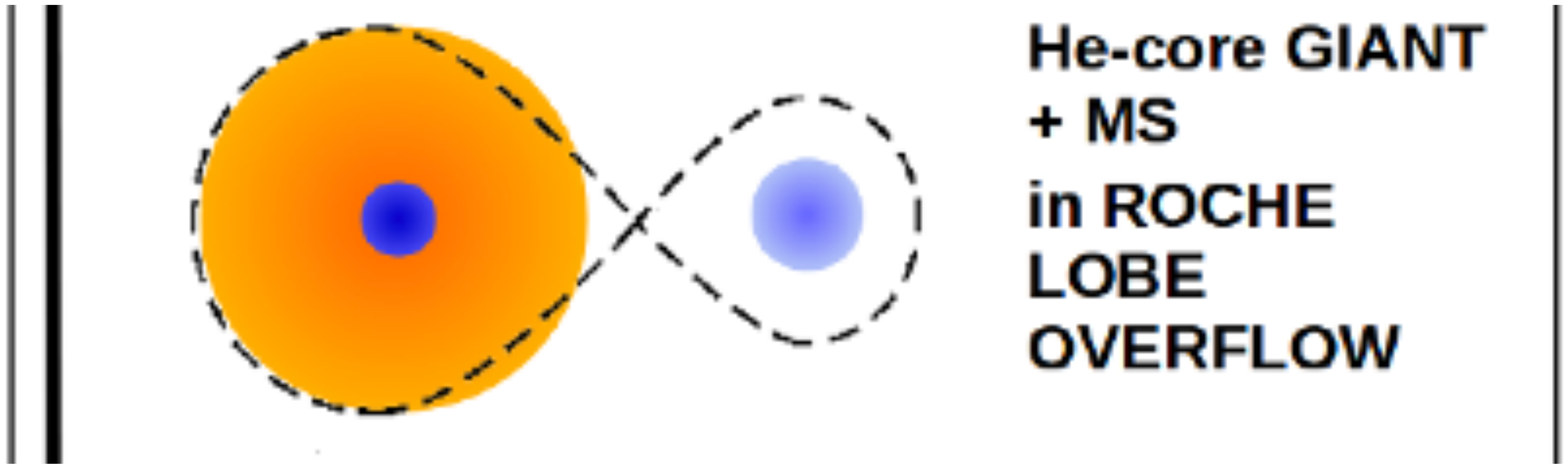
Credits: Michela Mapelli

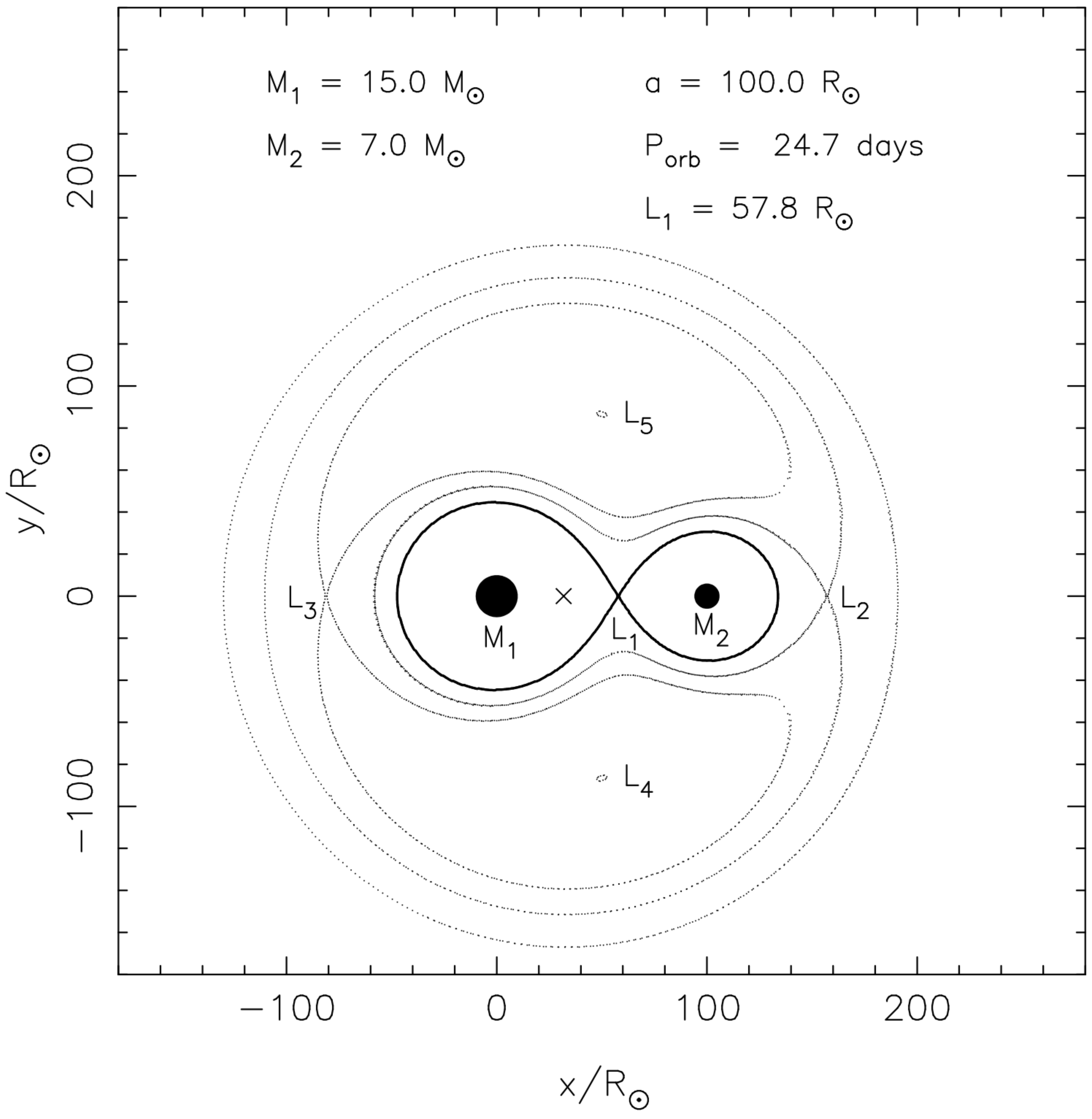
Mass transfer

- **Roche lobe:** $r_{\rm RL} = a\dfrac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}$

- **Mass transfer via Roche-lobe overflow:** mass can be transferred to the other object via L1

- **Orbit shrinks** in case of **stable** mass transfer



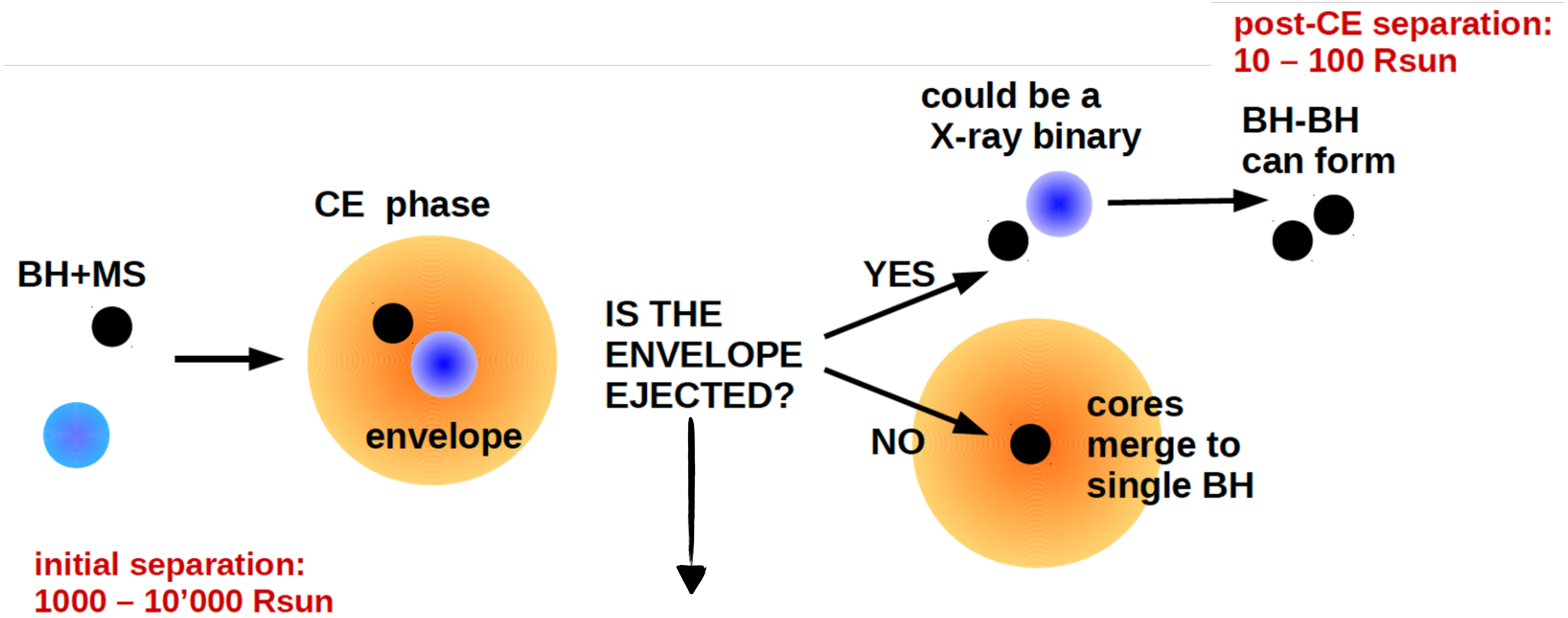


Credits: Michela Mapelli

Refs: Tauris and Van den Heuvel 2003

Common Envelope

See here a short movie describing the CE phase

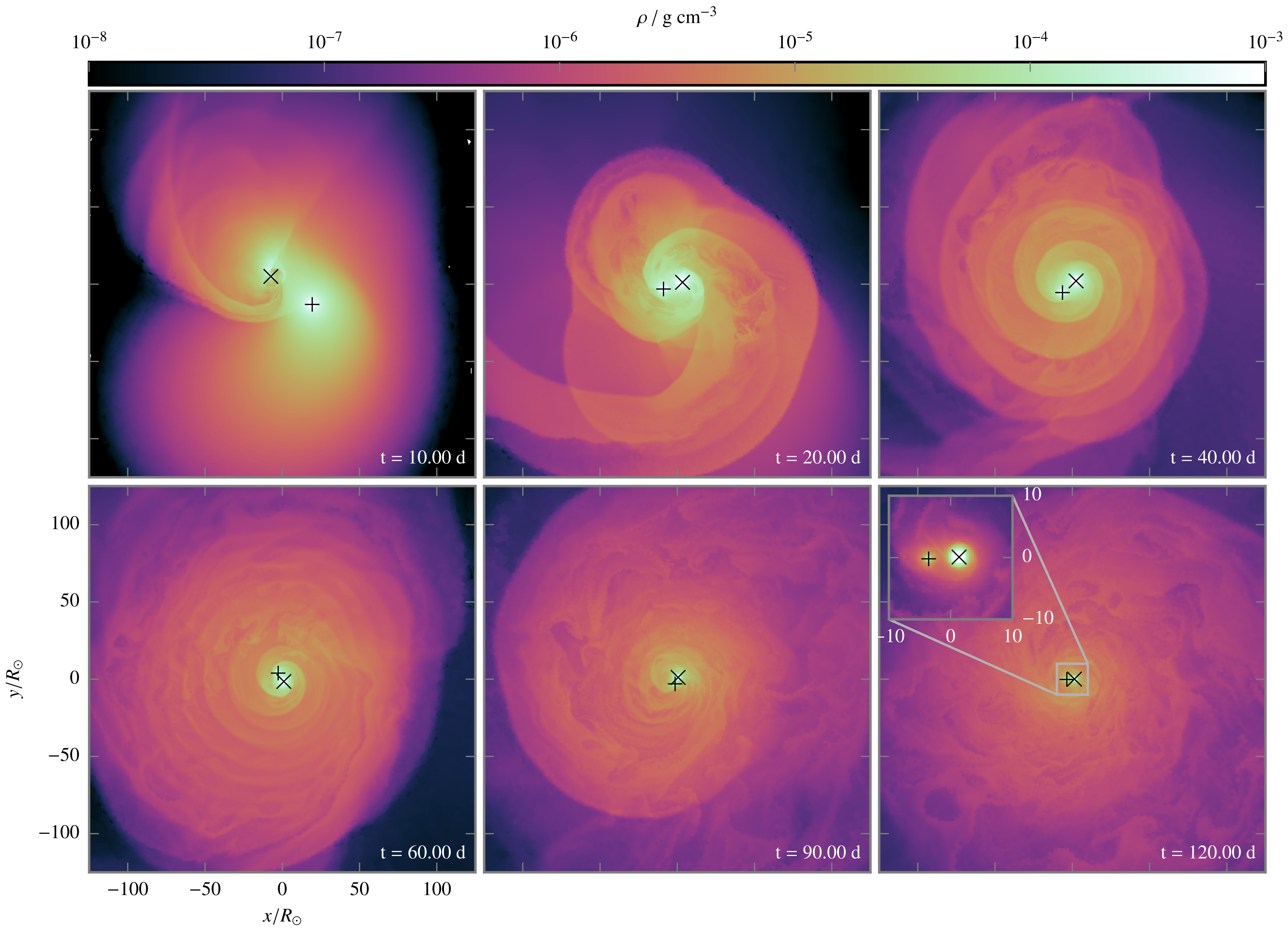


This is the most important question.
i.e., does the binary survive the CE phase?

Credits: Michela Mapelli

Modelling the common envelope

Hydrodynamical simulations



$\alpha\lambda-$formalism

- $$E_{\mathrm{bind},ini} = -\frac{G}{\lambda}\left(\frac{M_1 M_{1,\mathrm{env}}}{r_1} + \frac{M_2 M_{2,\mathrm{env}}}{r_2}\right)$$
- $$E_{\mathrm{orb},ini} = \frac{1}{2}\frac{G M_{c,1} M_{c,2}}{a_{ini}}$$
- $$E_{\mathrm{bind},ini} = \Delta E_{\mathrm{orb}} = \alpha(E_{\mathrm{orb},fin} - E_{\mathrm{orb},ini})$$

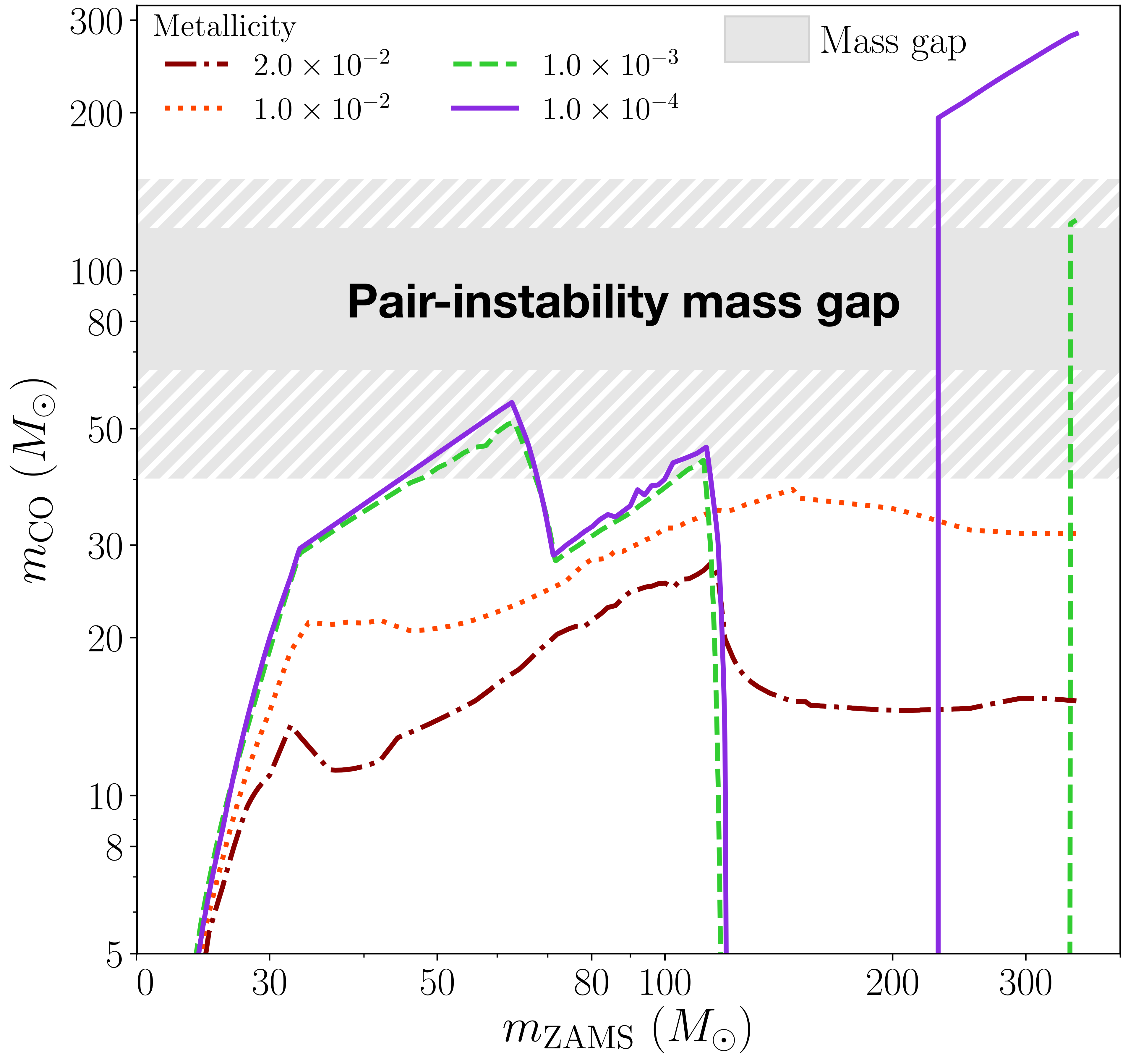
Credits: Ohlmann et al. 2016

Refs: Webbink 1984, Hurley et al. 2002

Can we explain all GW observations with only the isolated formation channel ?

Pair-instability Super Nova

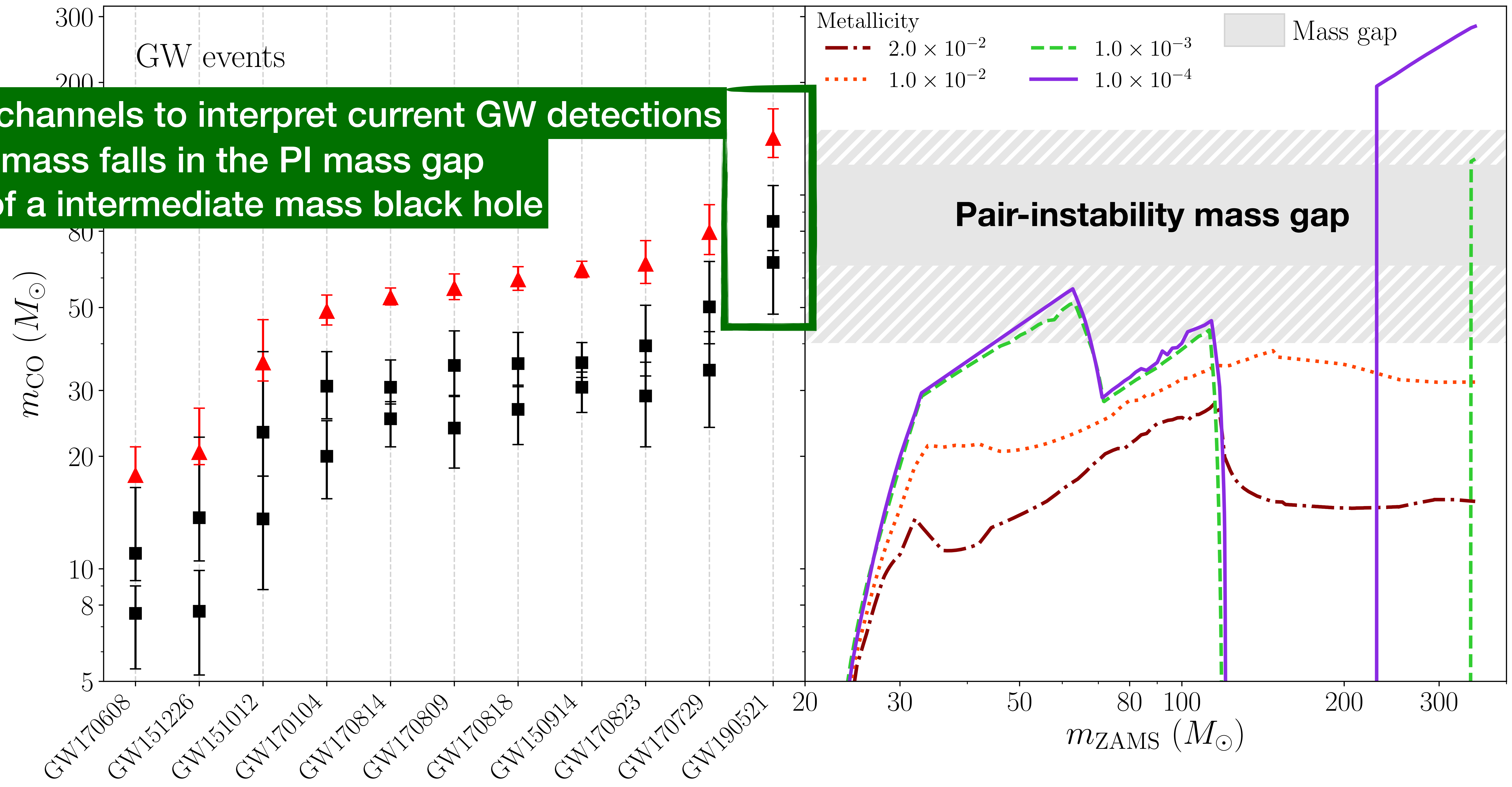
- Very massive stars ($M_{\mathrm{He}} \gtrsim 64\ \mathrm{M}_{\odot}$)
- Central temperature $T > 7 \times 10^{8}\ \mathrm{K}$
- Efficient production of gamma ray radiation in the core
- Gamma-ray photons scattering by atomic nucleus produce electron-positron **pairs**
- **Missing radiation pressure** produces dramatic **instability** and collapse, leaving no remnants ➡️



Refs: Spera and Mapelli 2017

Credits: LIGO-Virgo-KAGRA collaboration

Pair-instability Super Nova



We need other formation channels to interpret current GW detections

1. Primary mass falls in the PI mass gap
2. Formation of a intermediate mass black hole

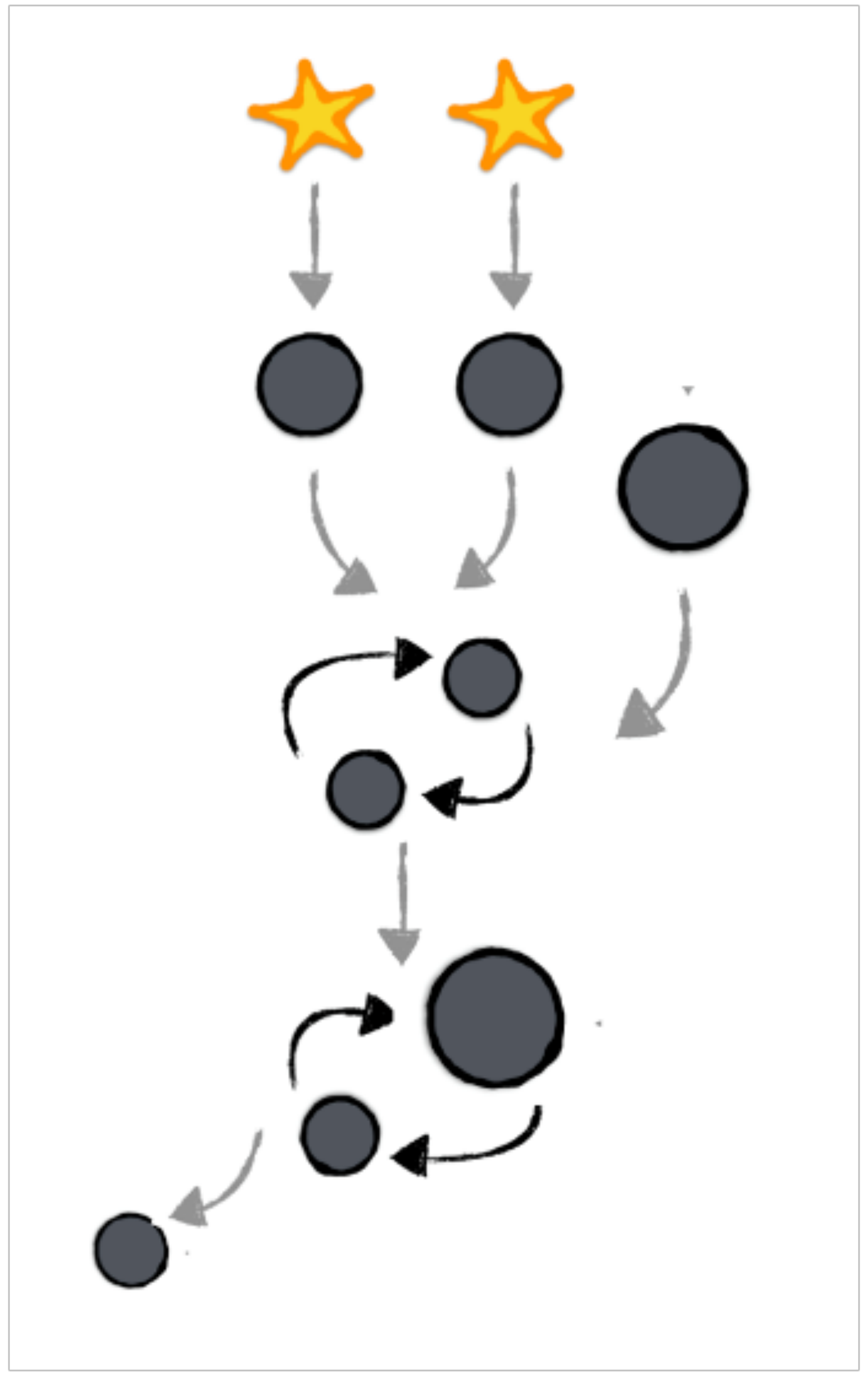
Credits: LIGO-Virgo-KAGRA collaboration

Dynamical formation channel

- Compact objects form and evolve with dynamical processes

- Dynamical processes have effect only with $\rho > 10^3 \ \mathrm{stars\ pc^{-3}}$ (i.e. **Globular Clusters**, **Nuclear Star Cluster**, **Young Star Clusters**)

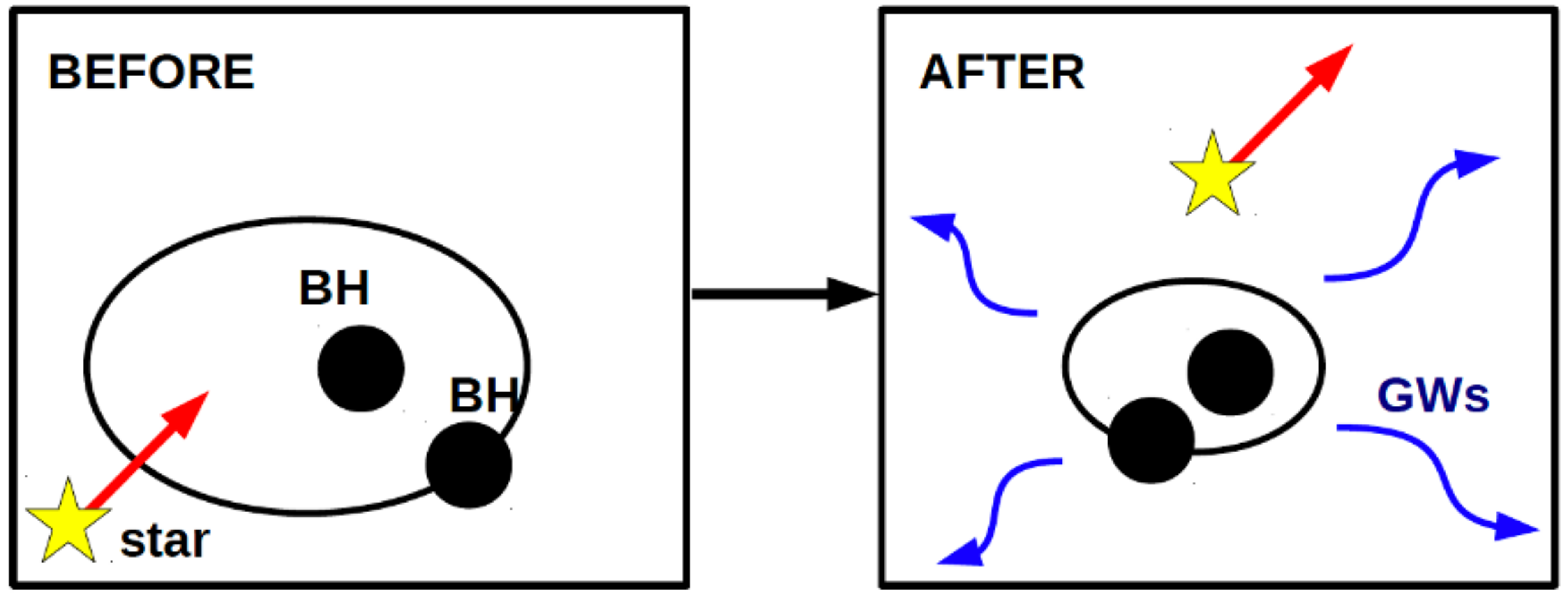
- Star clusters are also an active environment of formation of massive stars



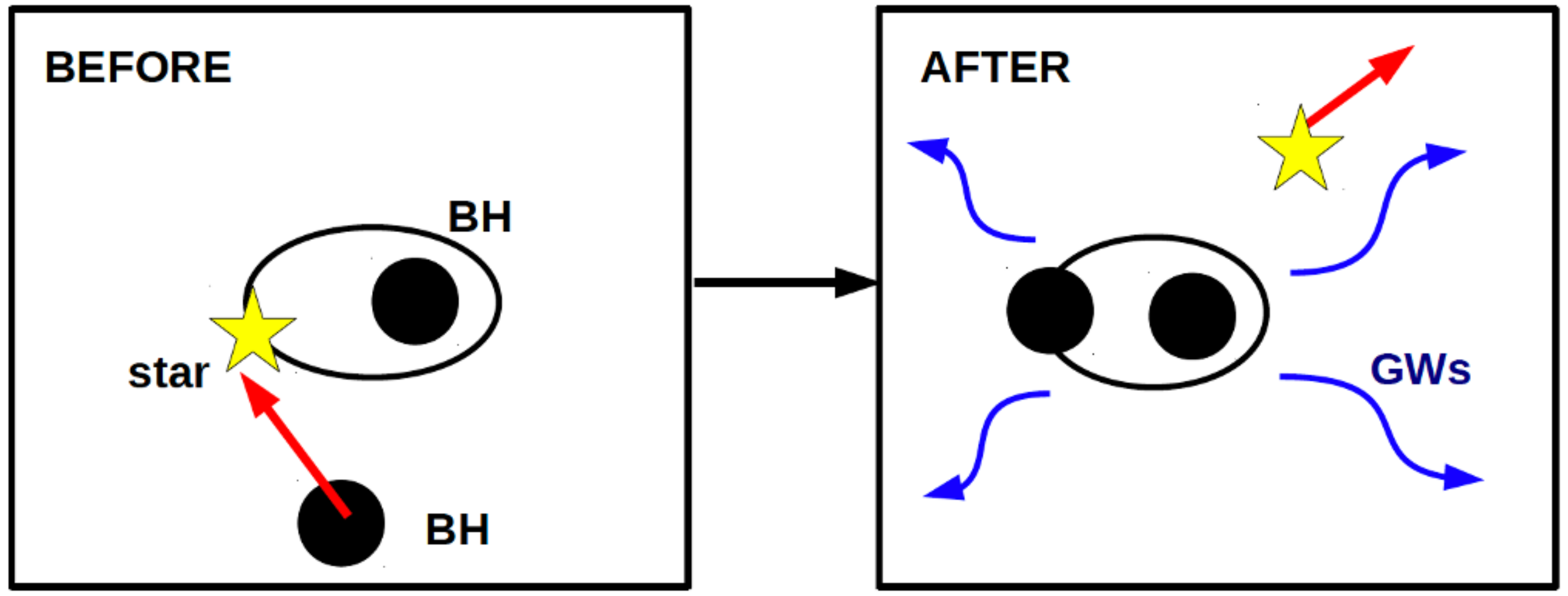
Refs: Lada and Lada 2003, Portegies Zwart 2010

Dynamical formation channel: processes

Hardening: star gains kinetic energy from the binary system ➡️ binary system shrinks



Credits: Michela Mapelli



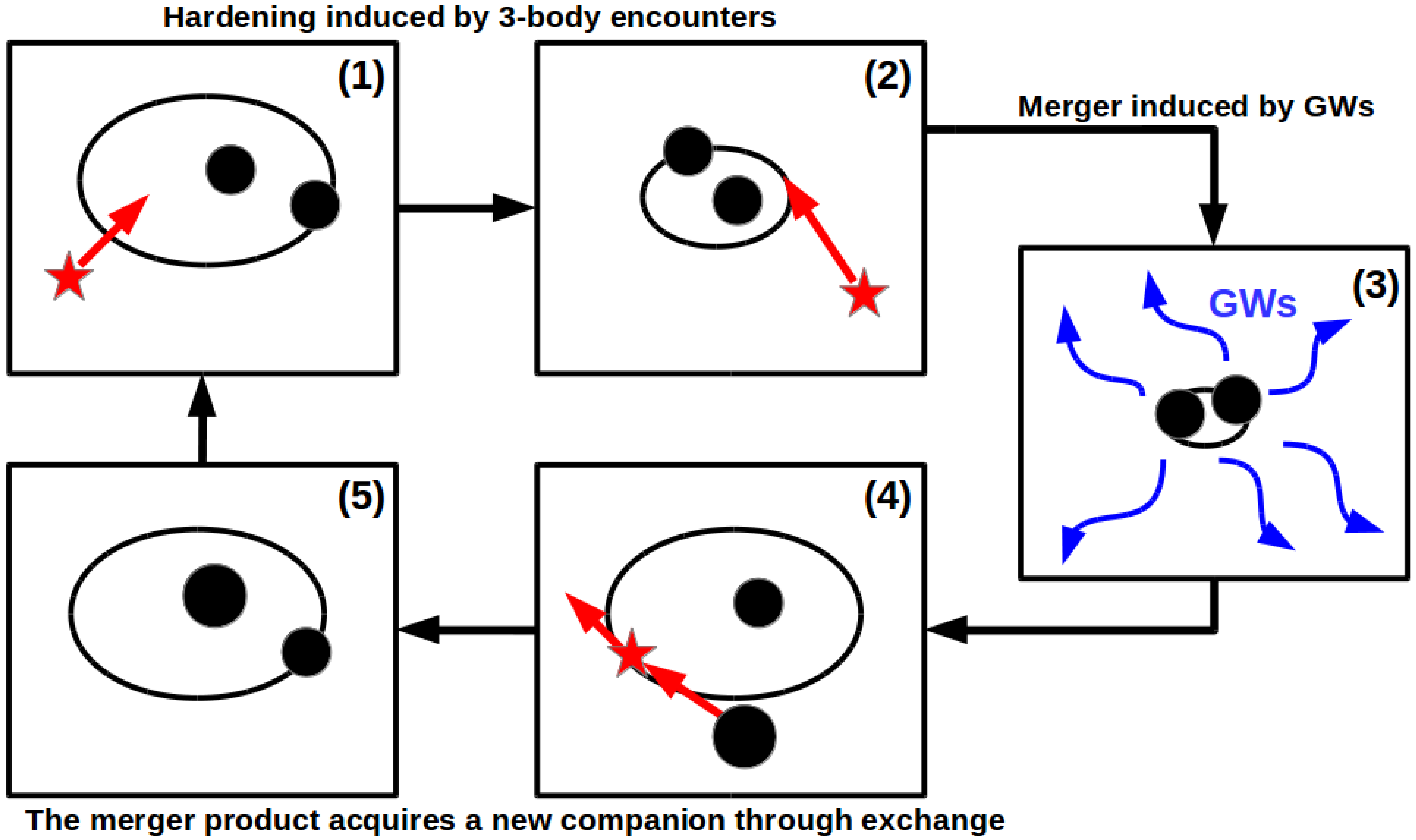
Credits: Michela Mapelli

Exchange: making single BH part of a binary systems ➡️ merger of massive BH with misaligned spins

Refs: Ziosi 2014, Mapelli 2018

Dynamical formation channel: processes

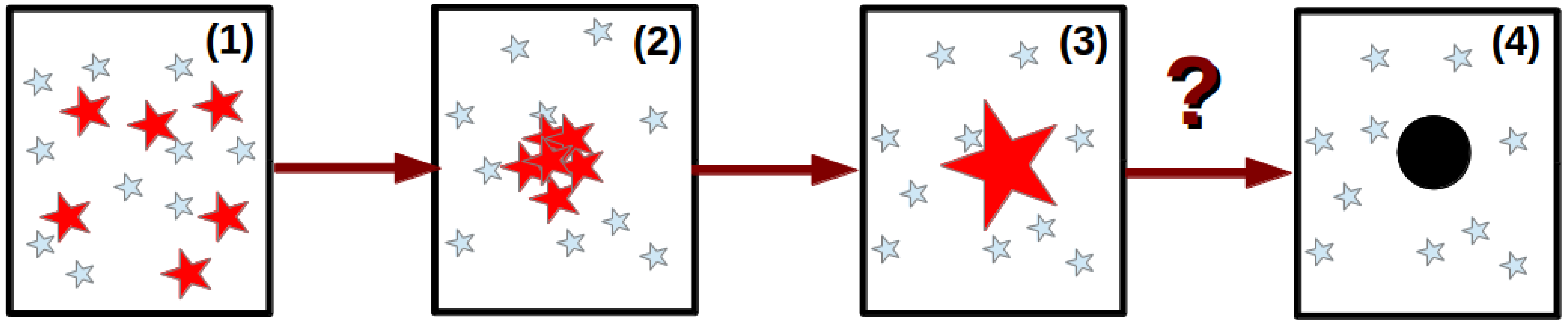
Hierarchical mergers: Merger remnant can become part of a binary by exchange ➡️ BH can grow in mass because of repeated mergers



Credits: Michela Mapelli

Refs: Ziosi 2014, Mapelli 2018

Runaway collisions



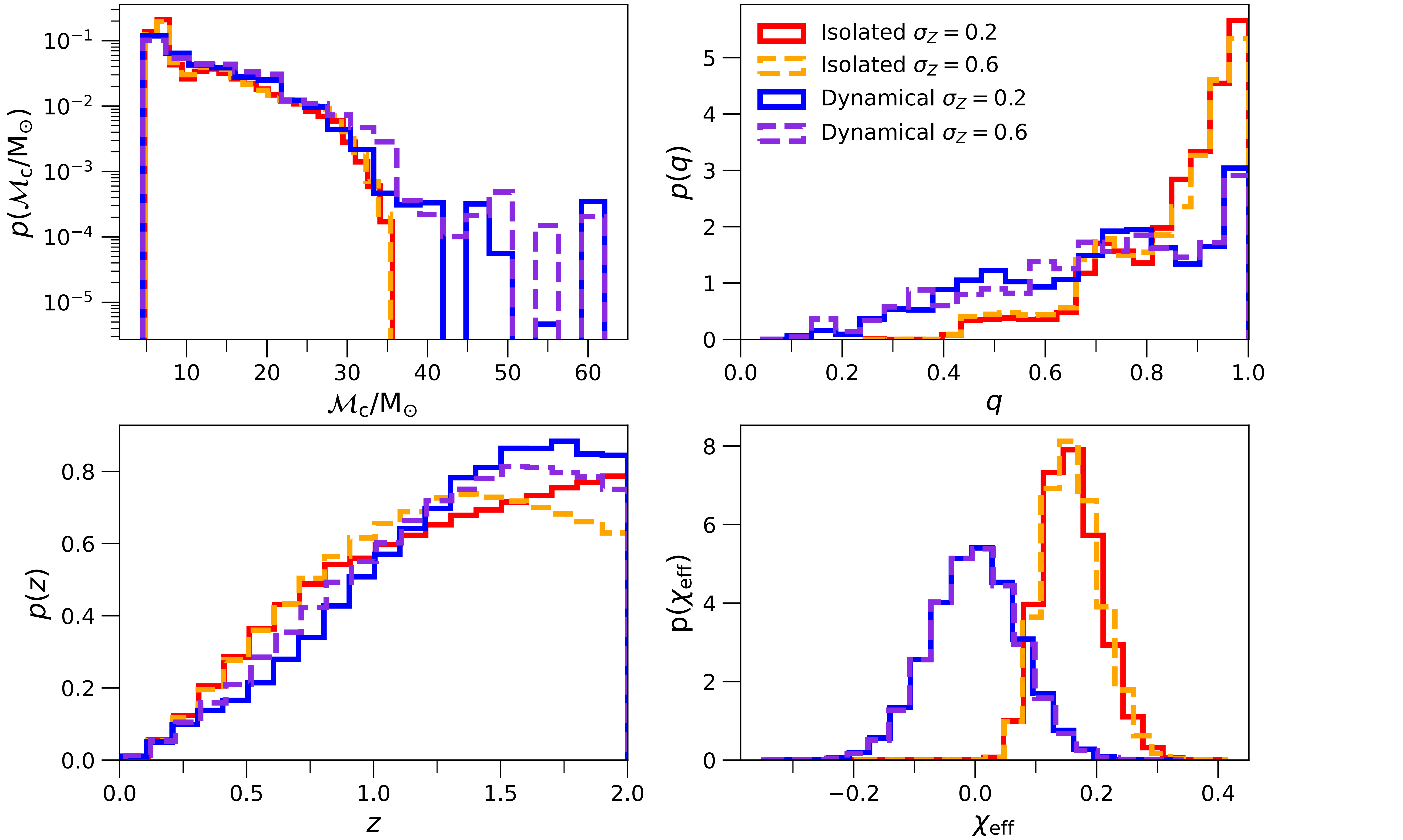
Credits: Michela Mapelli

- Mass segregation brings massive stars into the center
- Massive star collide, merge, and form super-massive stars capable of merging in the **pair-instability mass gap BH**

Refs: Mapelli 2018, Di Carlo et al. 2021

Is there a predominant formation channel?

Signatures

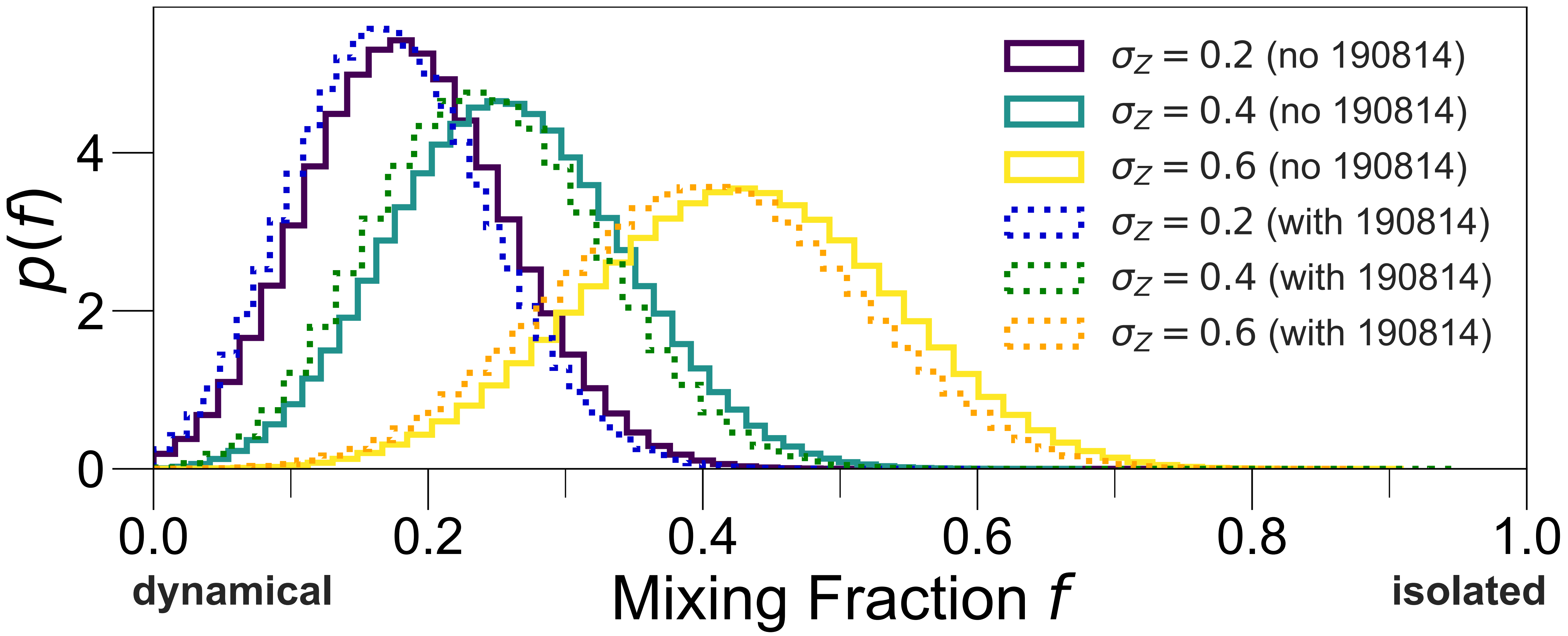


Refs: Bouffanais et al. 2021

Mixing fraction

$$\mathscr{L}(\{d\}\,|\,\Lambda) \propto \prod_{i}^{N_{obs}} \frac{\int \mathscr{L}(d_i\,|\,\theta)\pi(\theta\,|\,\Lambda)}{\xi(\Lambda)}$$

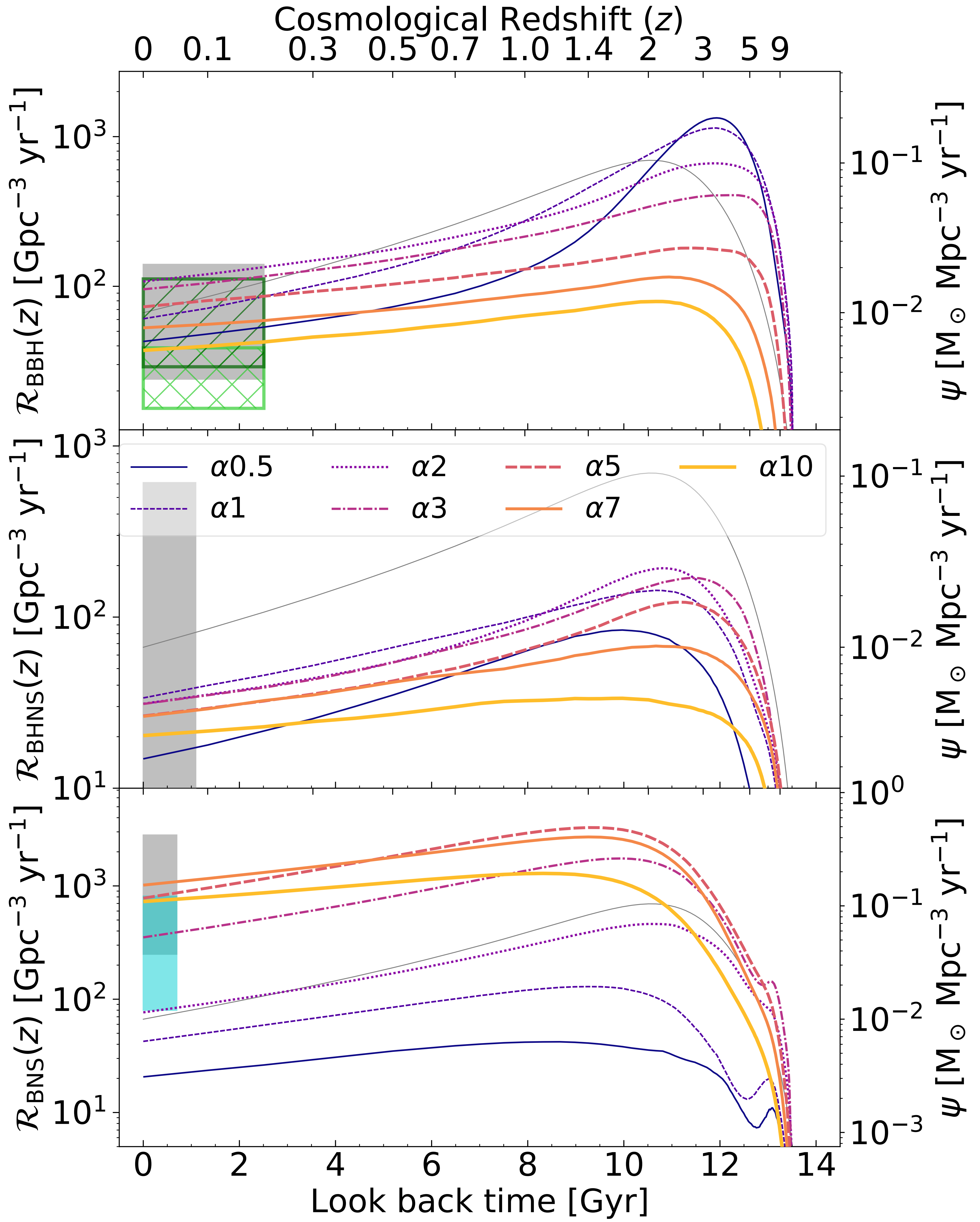
$$\pi(\theta\,|\,\Lambda) = f\,p(\theta\,|\,\mathrm{iso}, \sigma_z) + (1 - f)p(\theta\,|\,\mathrm{dyn}, \sigma_z)$$



Refs: Bouffanais et al. 2021, Zevin et al. 2021

Today's hands on

- `cosmoRate`
- d_L (z, assuming a cosmology) is a waveform parameter
- A given population model must include a redshift distribution
- evaluate the merger rate density given a population of compact object mergers



Refs: Santoliquido et al. 2021

What you did (not) learn today

- Observed population properties
- Astrophysical processes driving the isolated and dynamical formation channels

Tomorrow

- Multimessenger Astrophysics
- Properties of host galaxies of compact object mergers

Further reading:

- This lecture is based on lecture materials from Marica Branchesi, Jan Harms, Tito Dal Canton, Michela Mapelli, Giuliano Iorio, Gaston Escobar, and Eleonora Loffredo

- References:

  - **Non-parametric models:** Mandel et al. 2017, Li et al. 2021, Rinaldi and Dal Pozzo 2022,

  - **Astrophysics of compact objects:** Mapelli 2018, Spera et al. 2022 (with dynamics), Costa et al. 2023 (and references therein)

- **See you this afternoon!**