Gravitational Wave Astrophysics

Lecture 1

Filippo Santoliquido
GSSI and INFN, L'Aquila, Italy

ICTP-SAIFR/IFT-UNESP, São Paulo, August 2024

In these lectures, you'll learn

1. Detectors, gravitational waves, data analysis
2. Population analysis and Astrophysics of compact objects
3. Multimessenger Astrophysics and host galaxies
4. Einstein Telescope and the future of GWs

You'll learn how to use

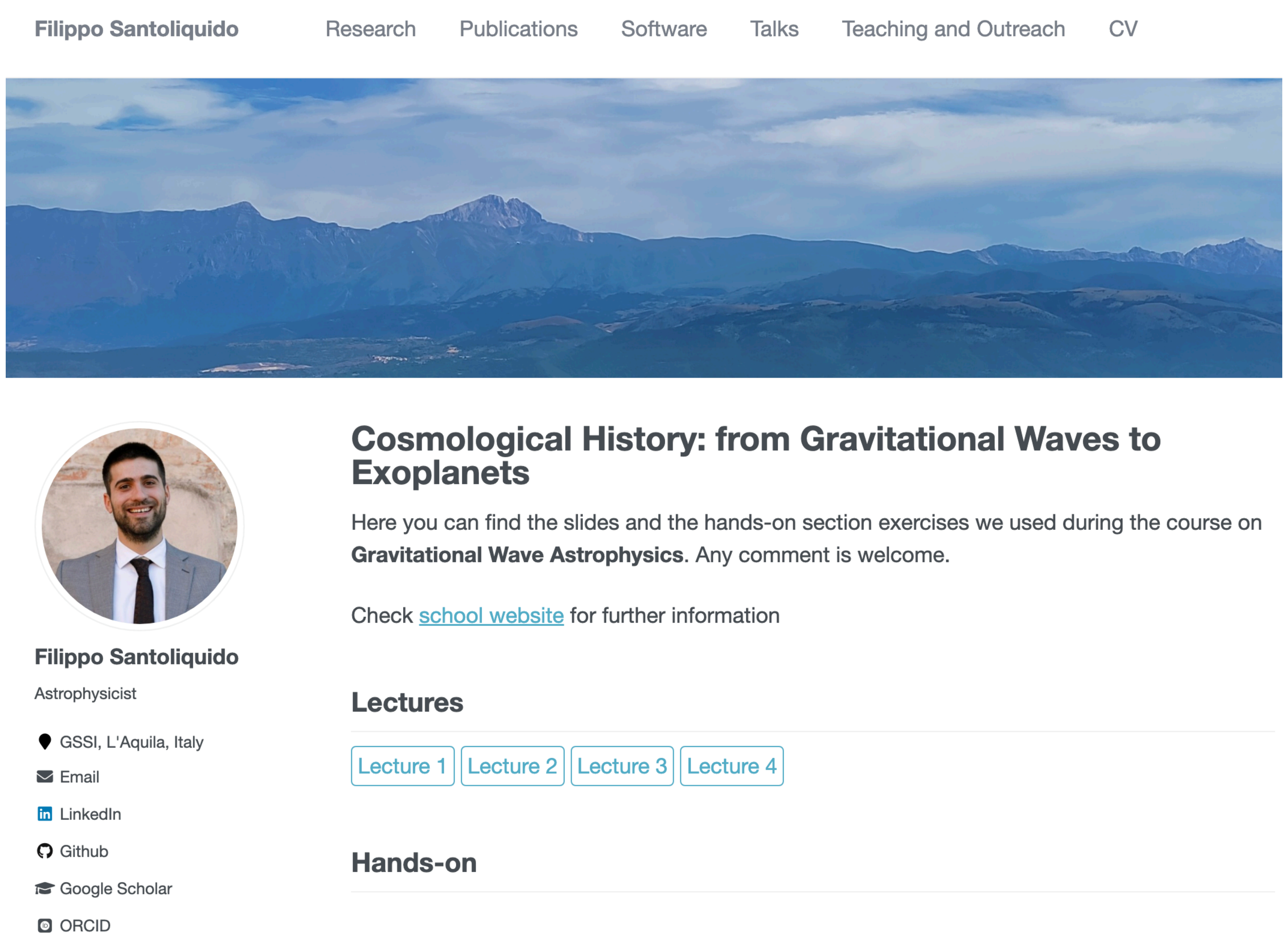
1. GWFish
2. cosmoRate

In this lecture, you’ll learn

- GW milestones
- Detection process
- Parameter estimation of single events

Where to find these slides

https://filippo-santoliquido.github.io/Brazil/



I introduce myself

Sofia Bisero

Elena Codazzo

Alessio Mei

Jacopo Tissino

Stefano Ascenzi

Andrea Cozzumbo

Eleonora Loffredo

Nandini Hazra

Lavinia Paiella

Samanta Macera

Marica Branchesi

Sara Gliorio

Soumen Koley

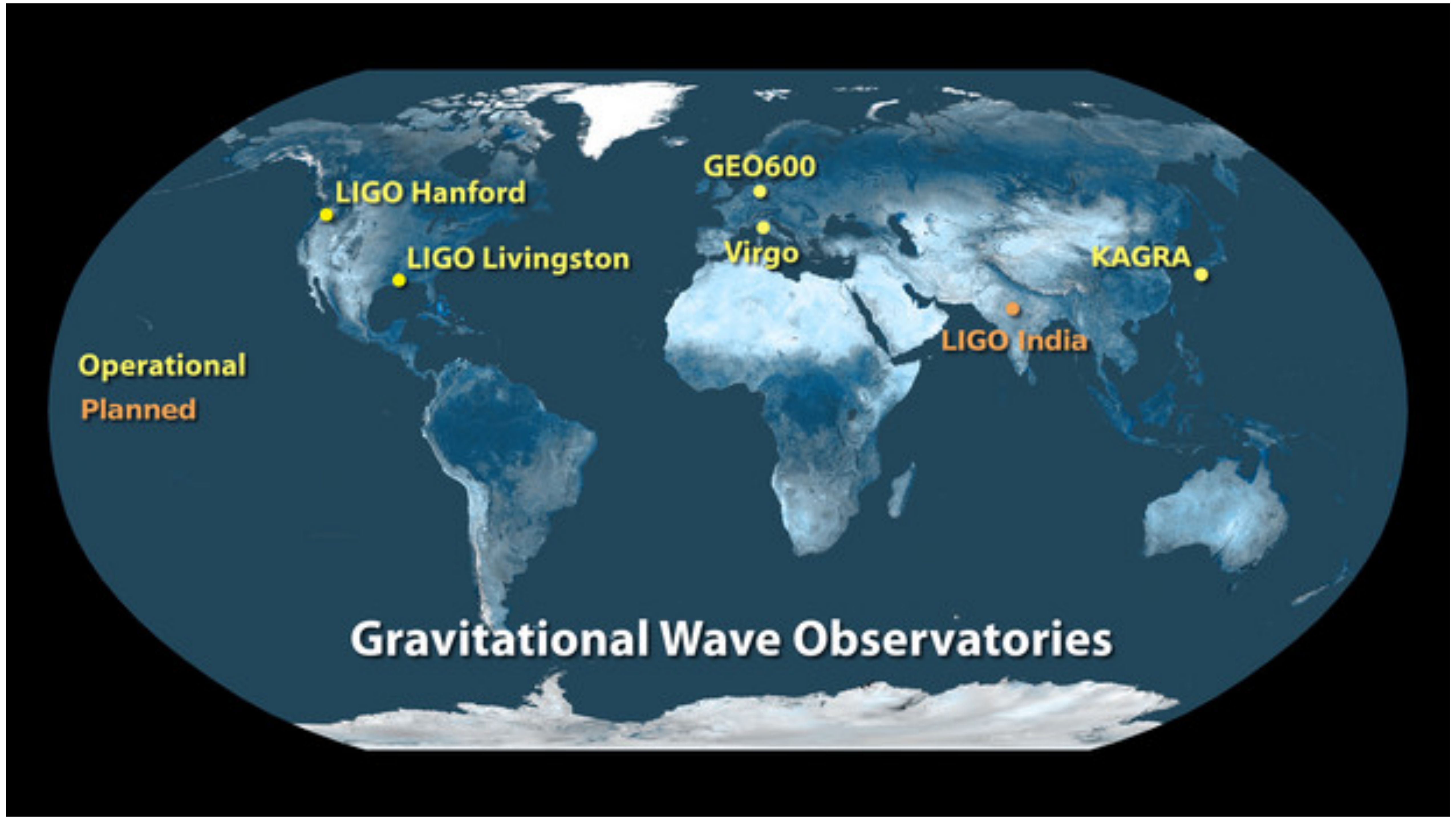
Annarita Ierardi

Jan Harms

And many others…

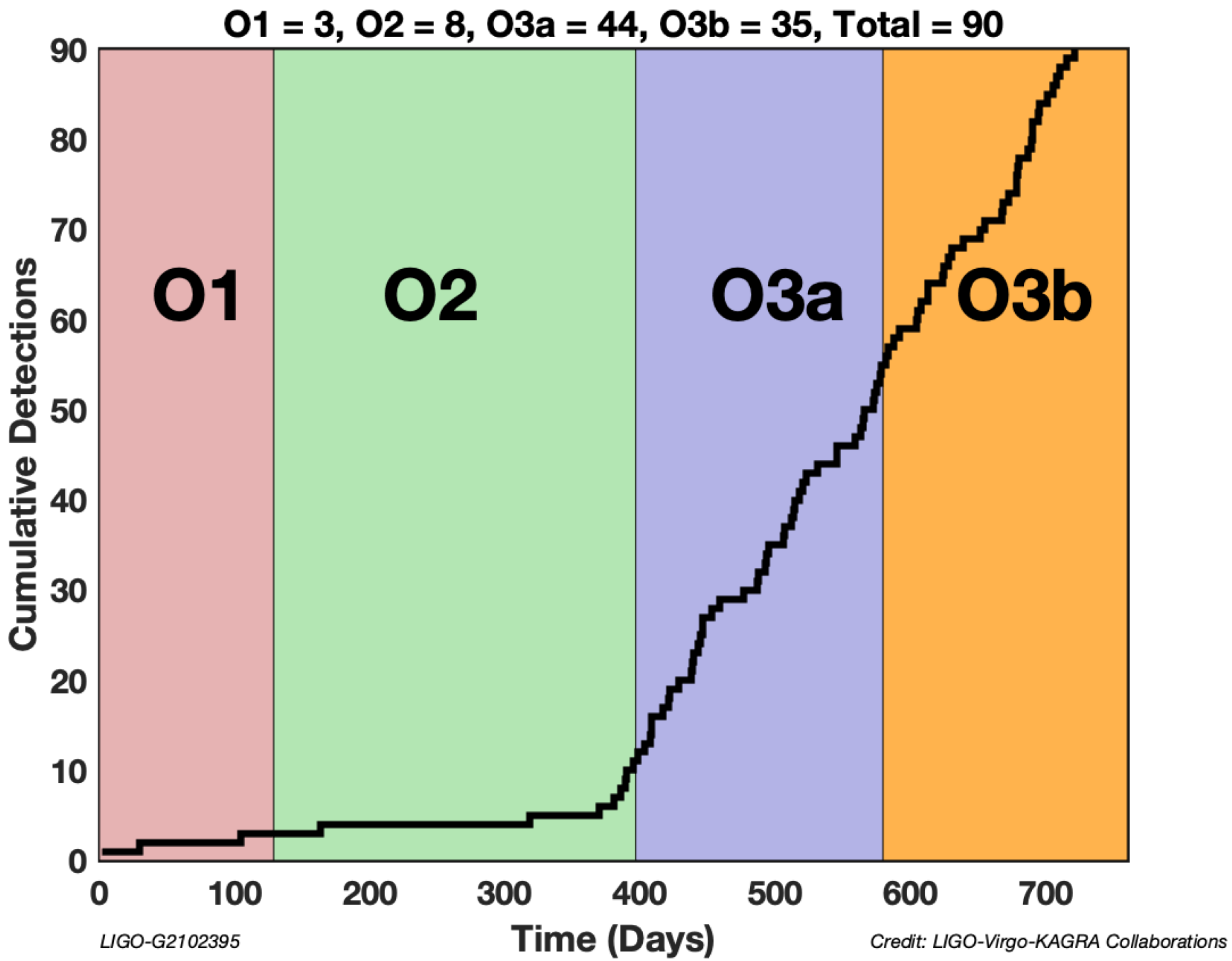
Contact me at filippo.santoliquido@gssi.it and ask questions, please

Detectors

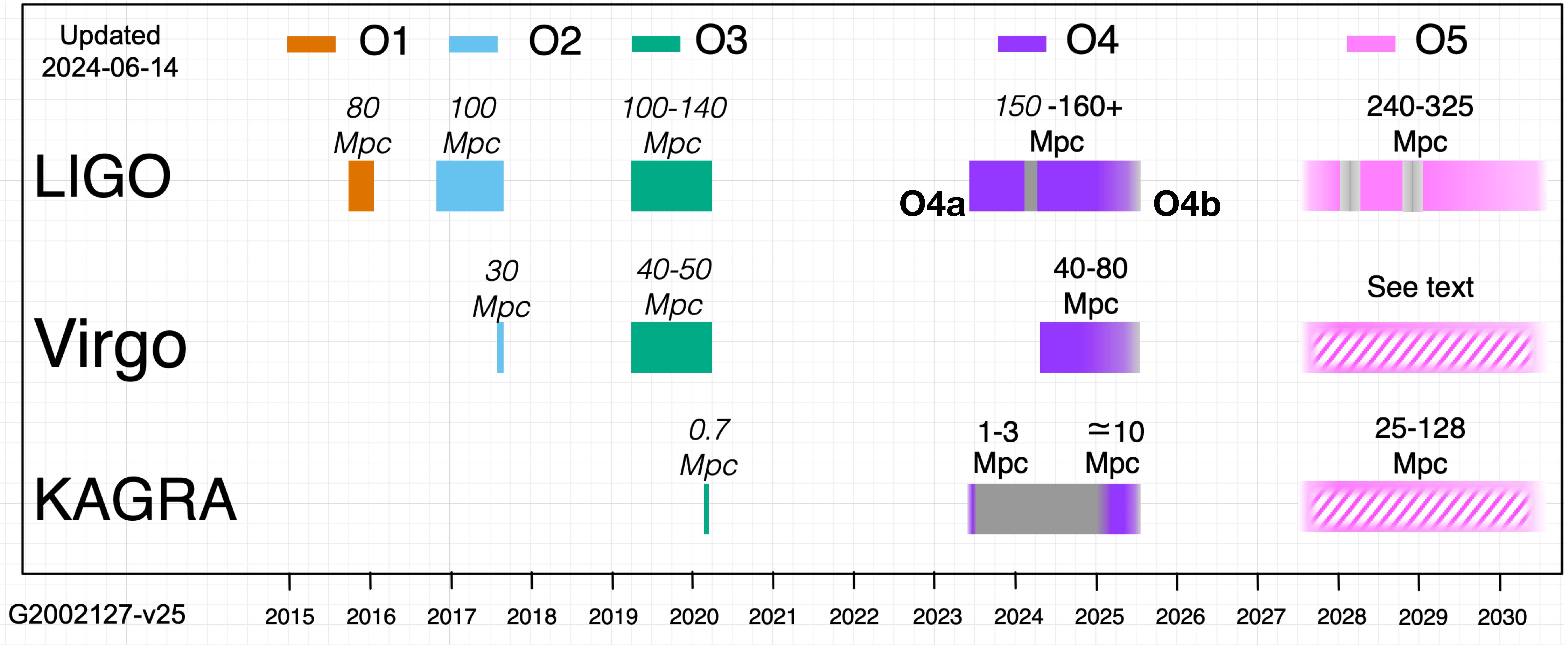


Credits: LIGO-Virgo-KAGRA collaboration

Observing runs



Ongoing observing run

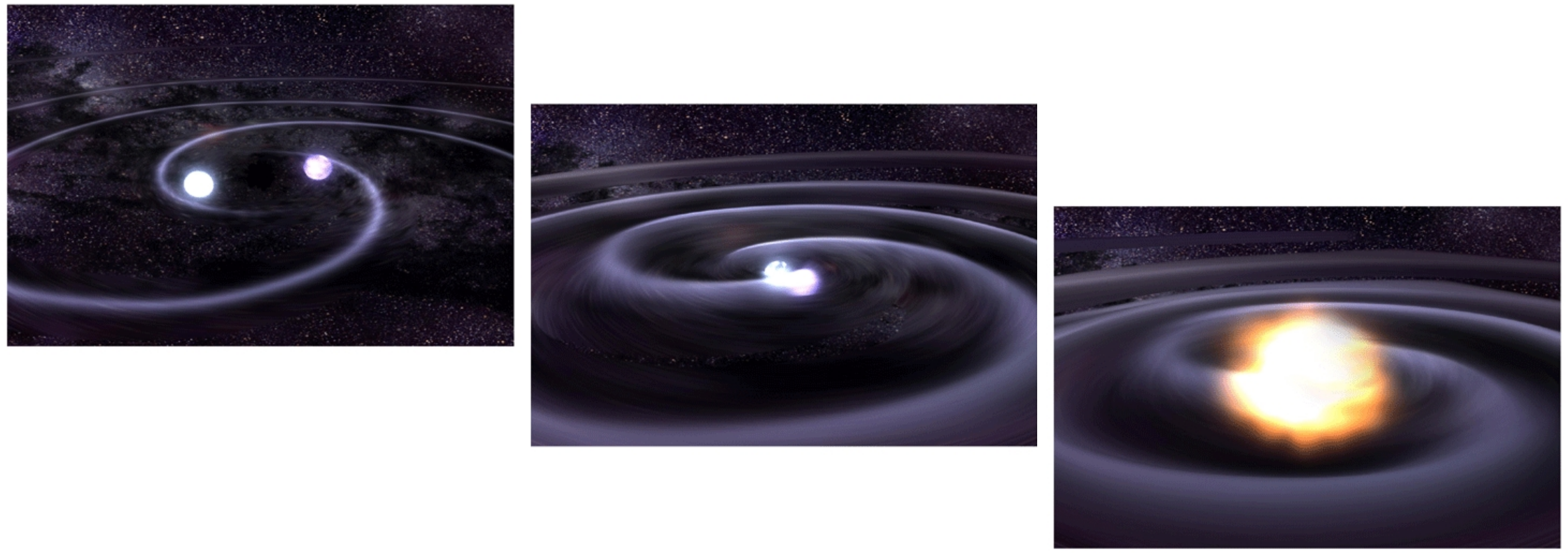


Credits: LIGO-Virgo-KAGRA collaboration, updates here

- *O4b will now end 9 June 2025.*
- *Public alerts*

Detected signals

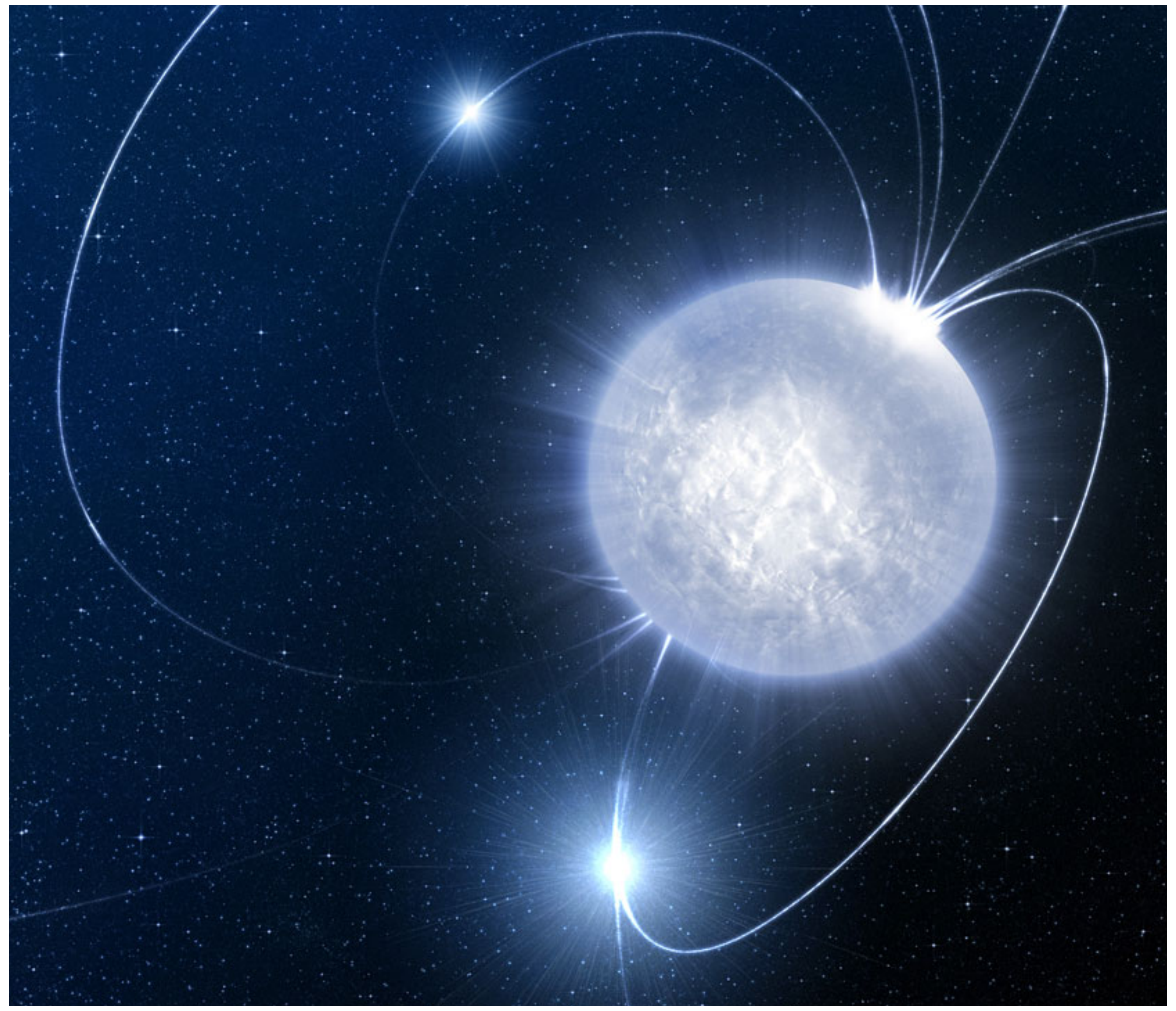
- CBC = compact binaries coalescence
  - Binary Black Holes (**BBHs**)
  - Binary Neutron Stars (**BNSs**)
  - Black hole-neutron star binaries (**BHNSs**)



Credits: LIGO-Virgo-KAGRA collaboration

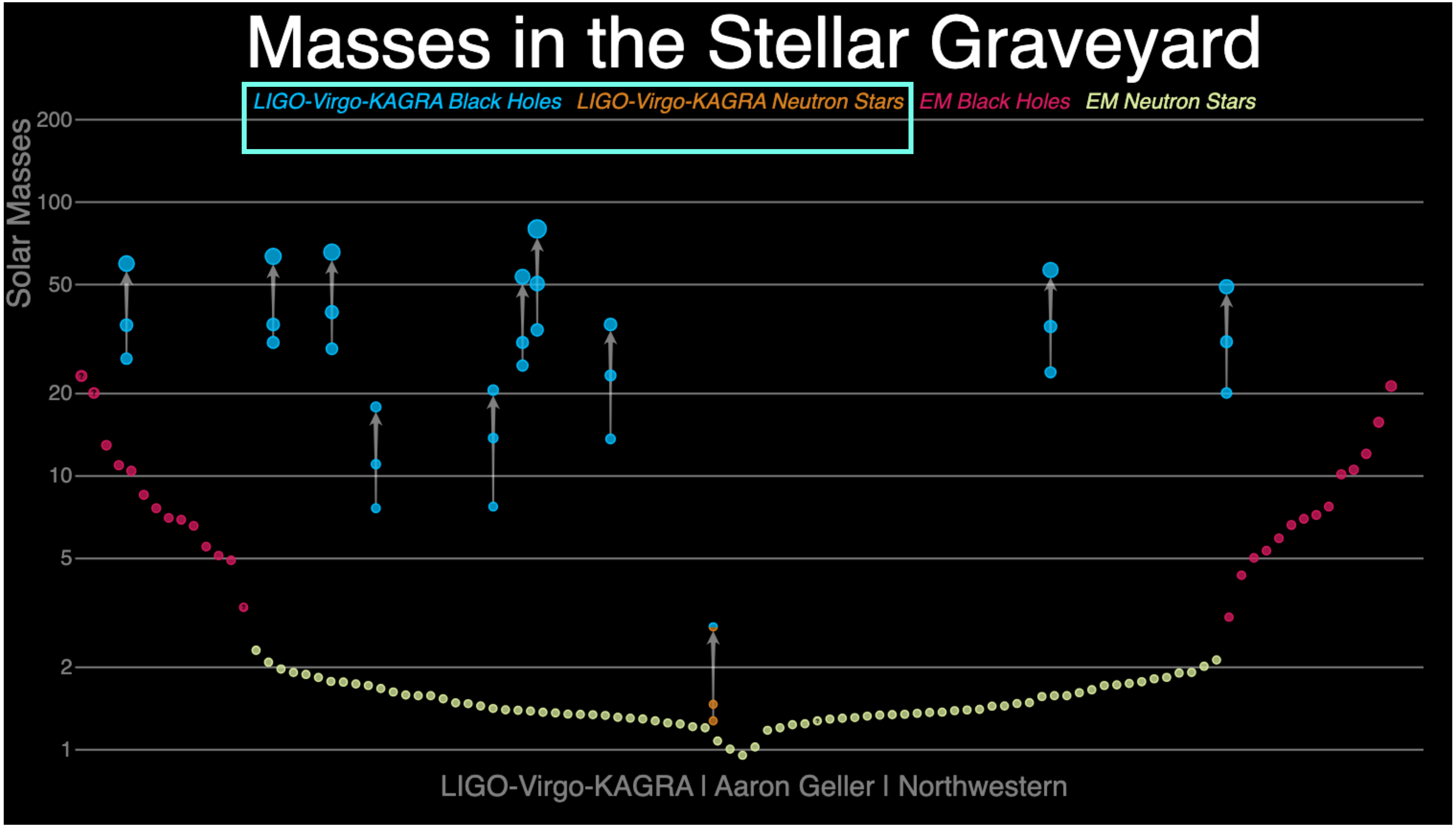
Detected signals

- CBC = compact binaries coalescence
  - Binary Black Holes (**BBHs**)
  - Binary Neutron Stars (**BNSs**)
  - Black hole-neutron star binaries (**BHNSs**)
- Other <u>(undetected)</u> sources:
  - short-duration burst
  - Continuous waves
  - Stochastic Background

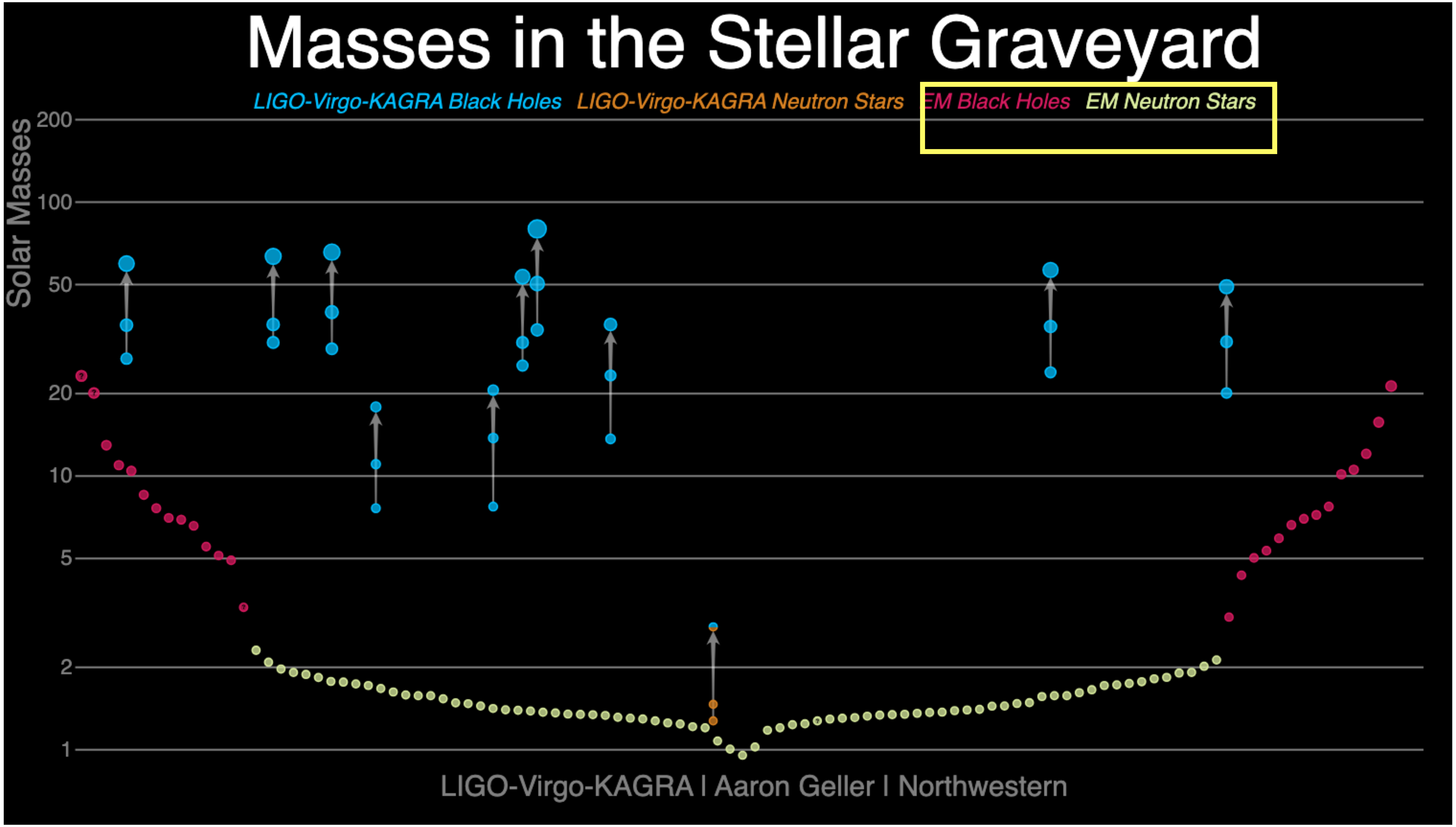


Credits: ESO

First Gravitational Wave Transient Catalog

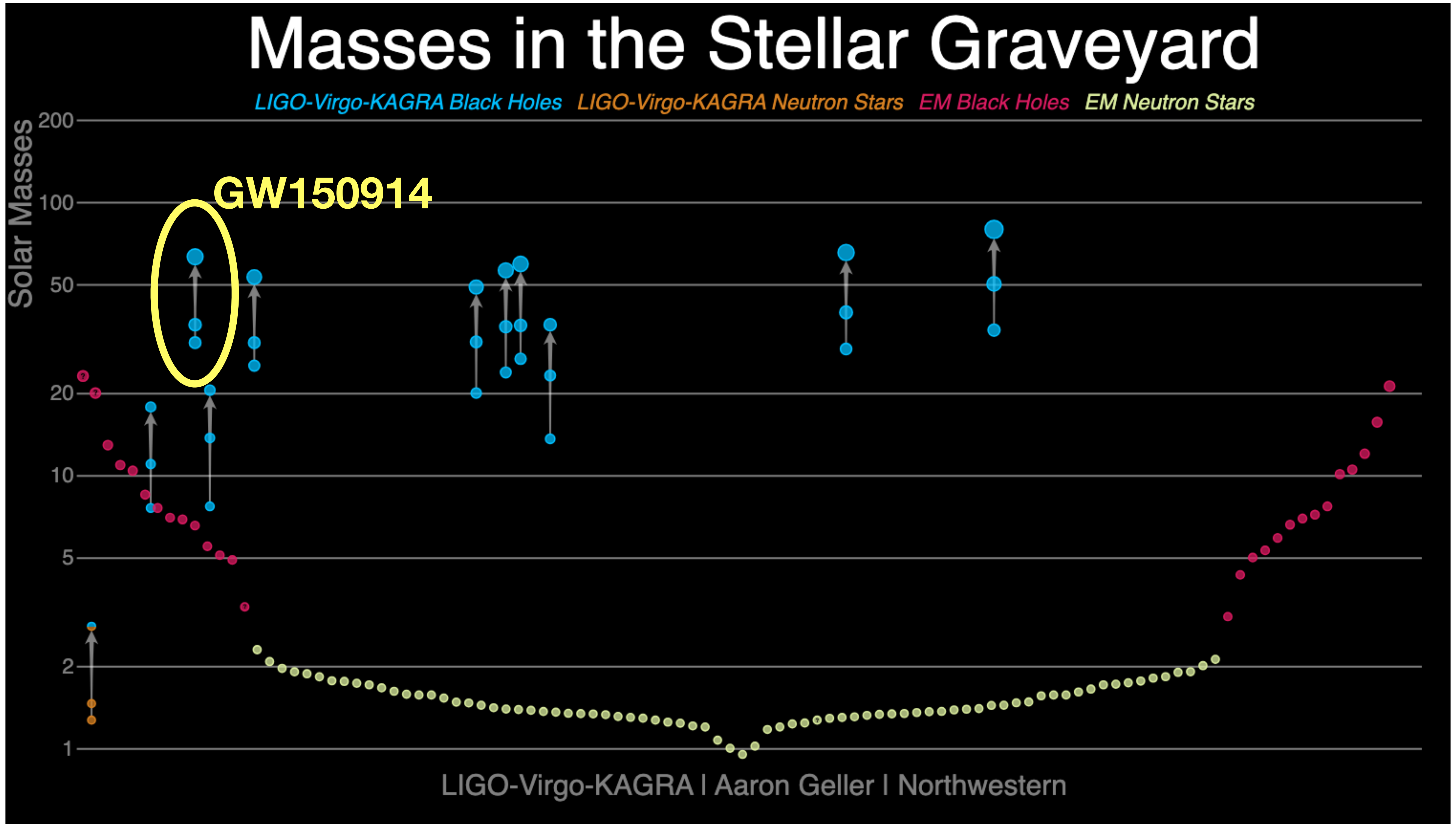


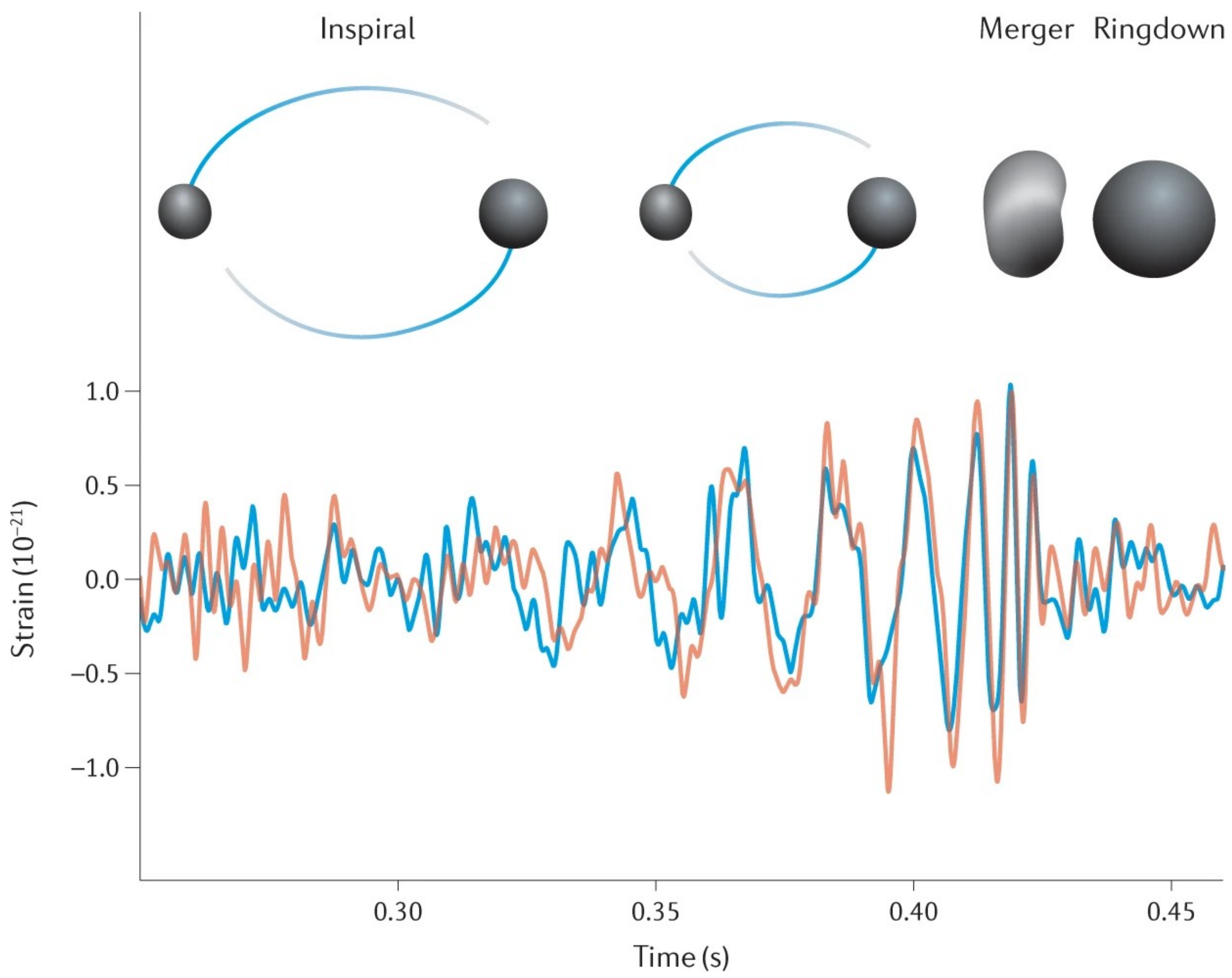
First Gravitational Wave Transient Catalog



GWTC-1

1st milestone: first ever detected direct GW signal

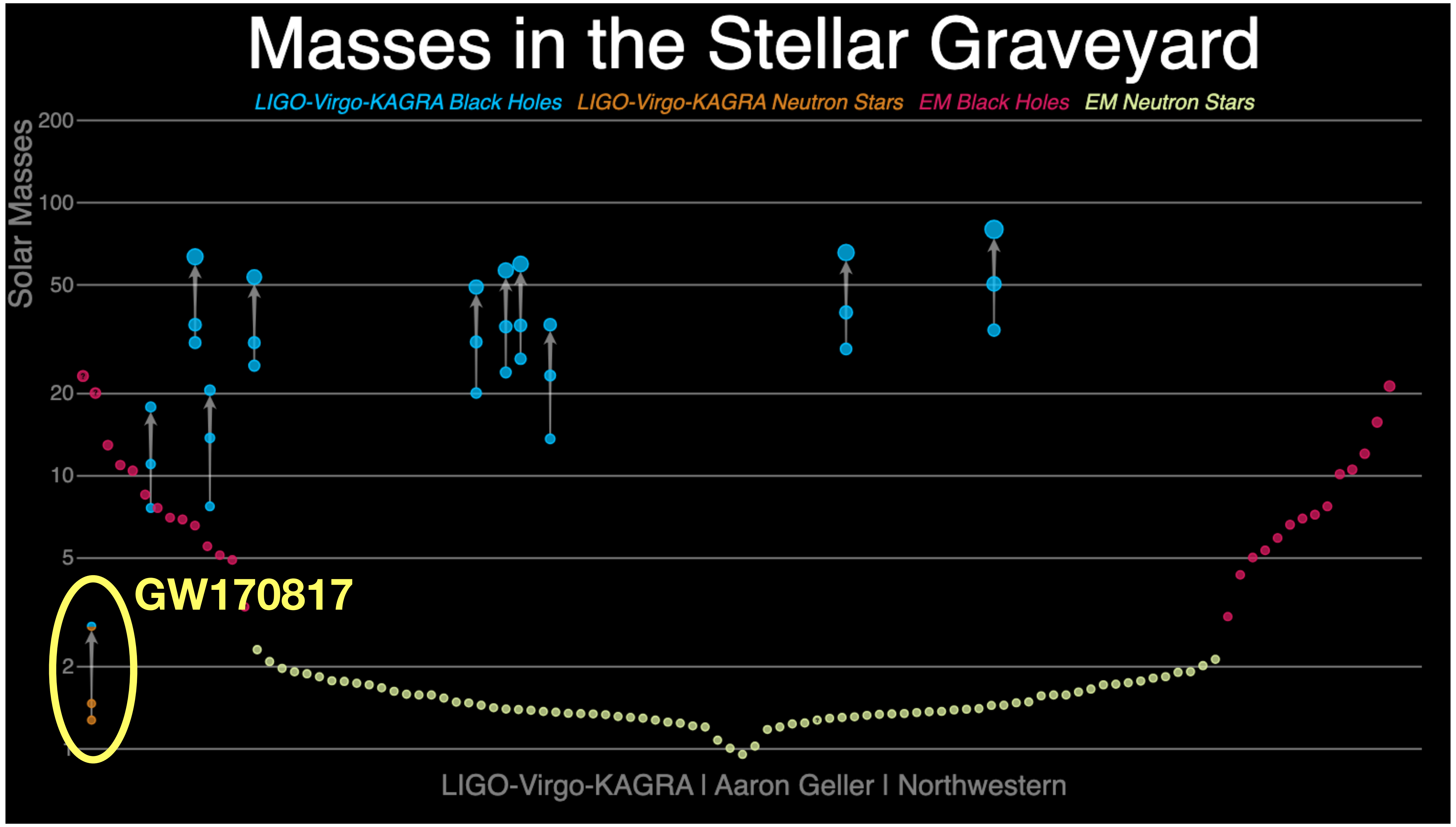


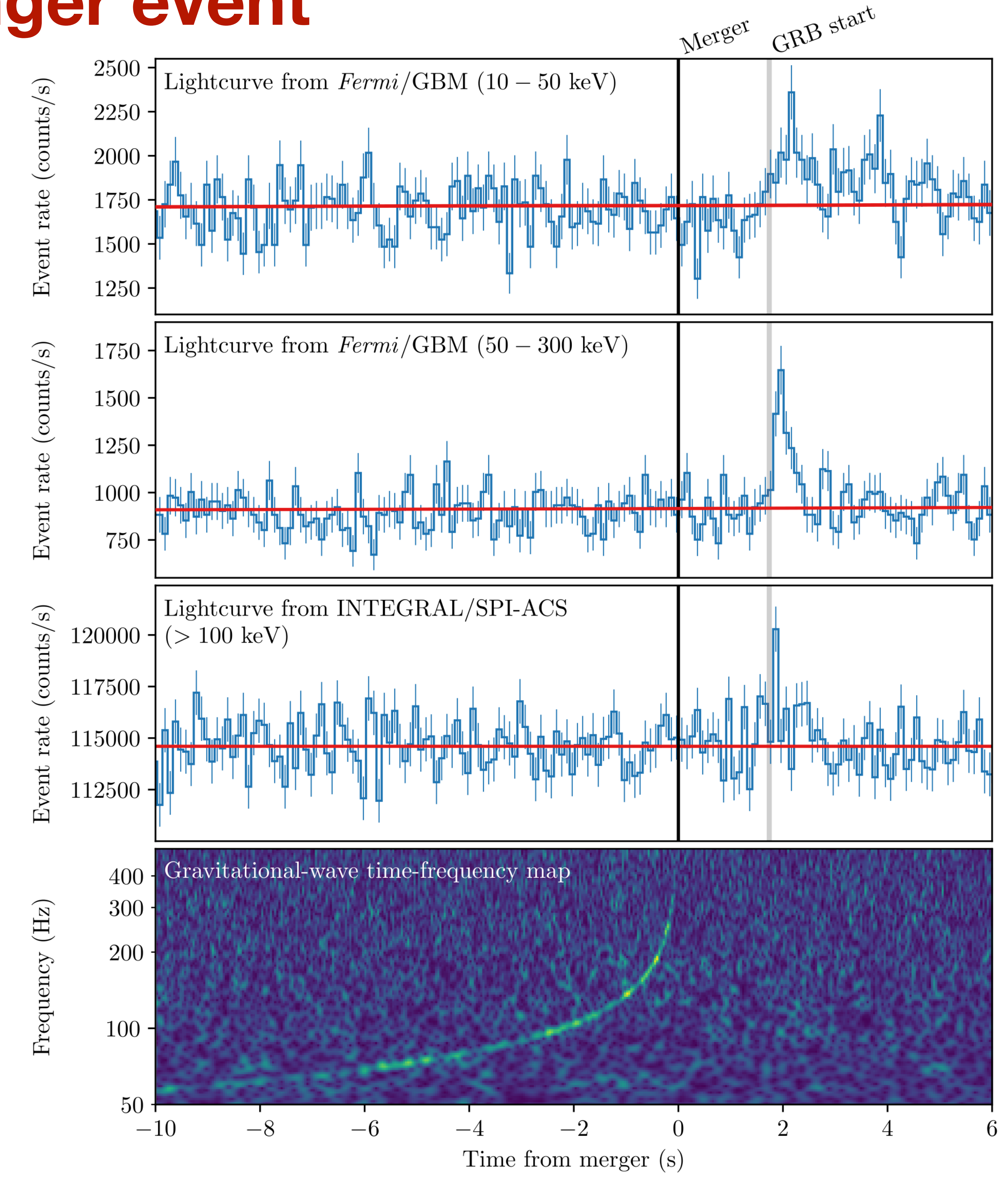


Credits: Bailes et al. 2021

GWTC-1

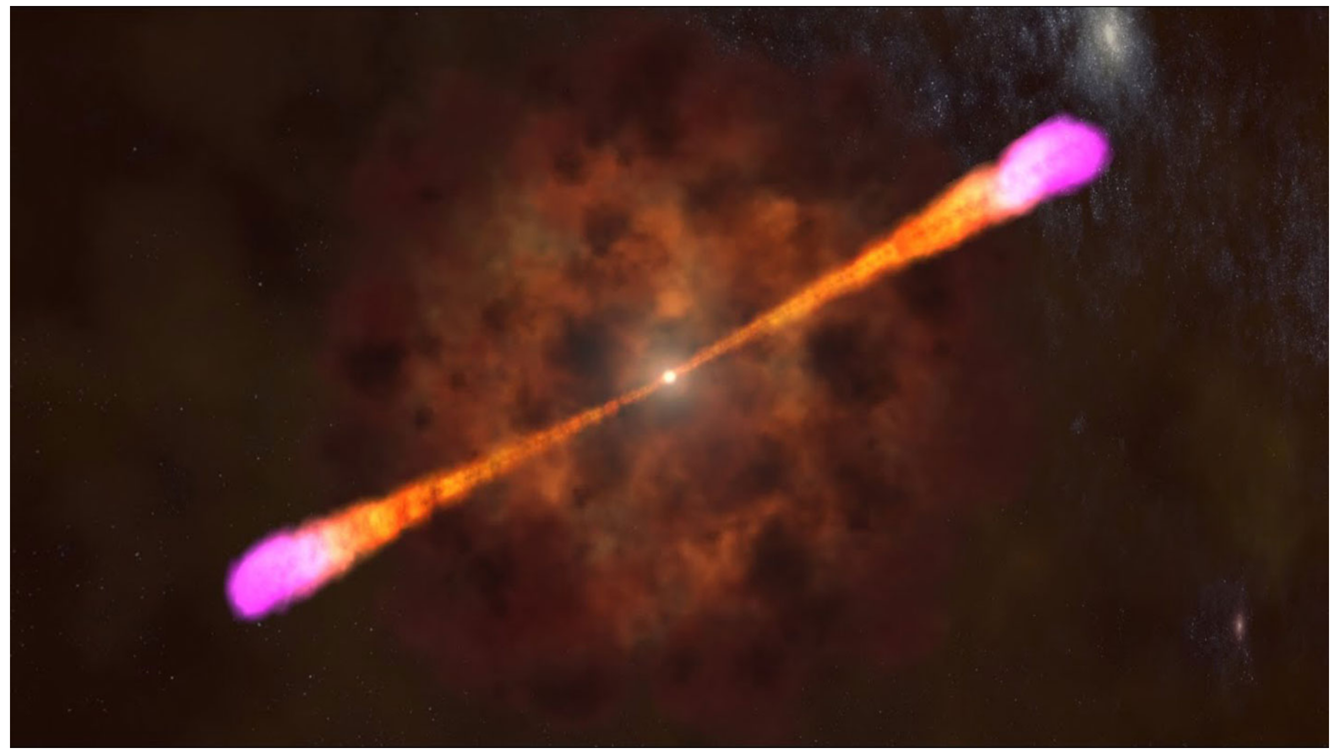
2nd milestone: first multimessenger event



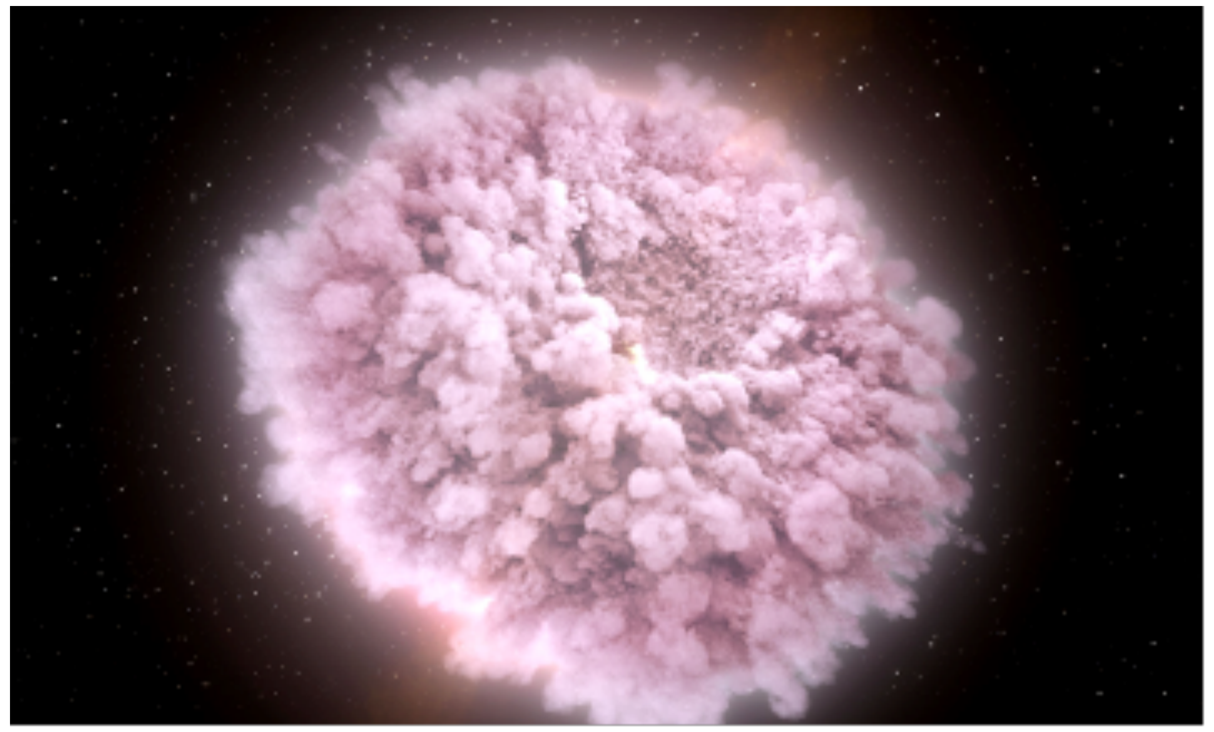


Credits: Abbott et al. 2017

Multimessenger astrophysics

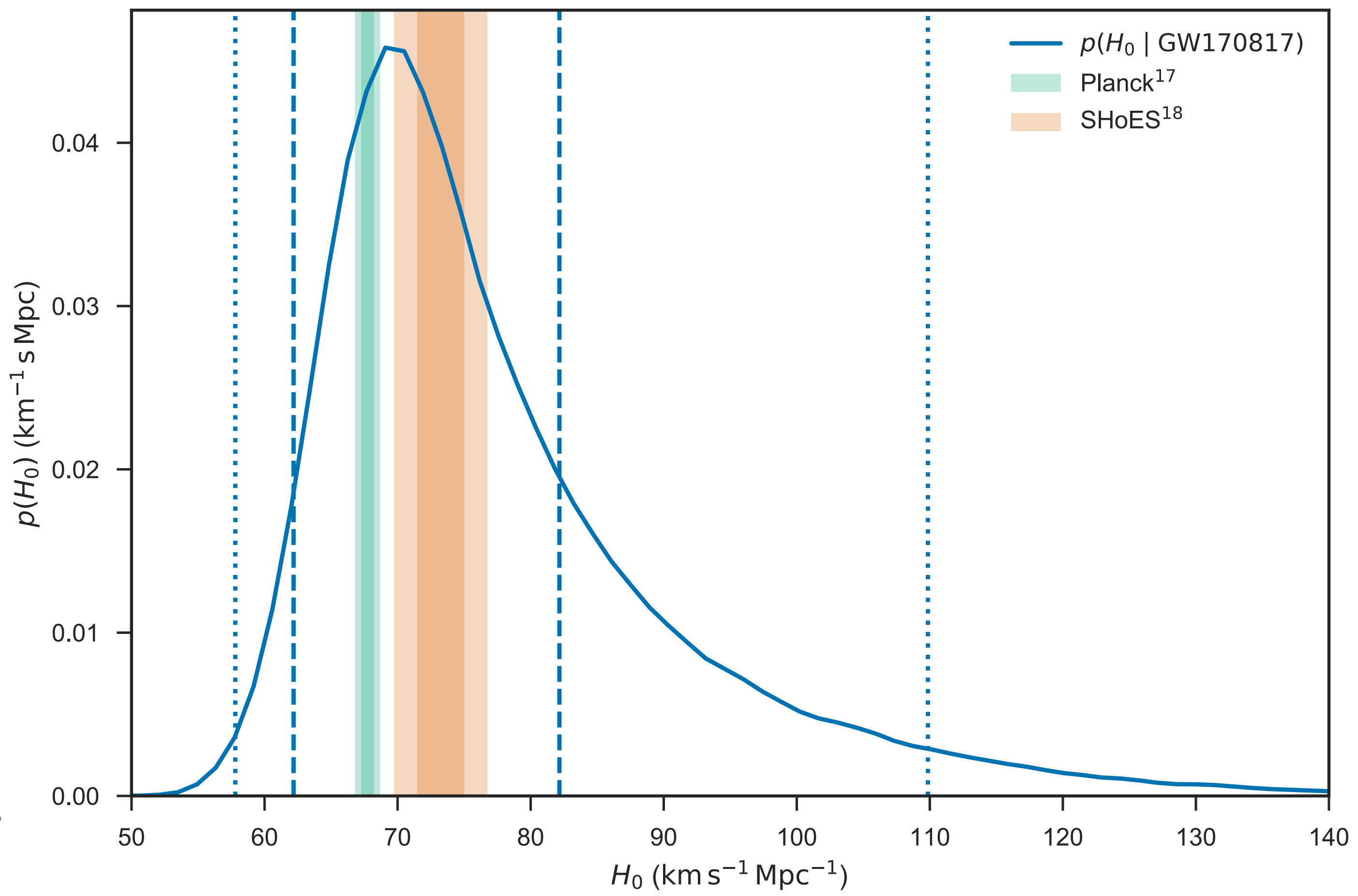


Credits: NASA



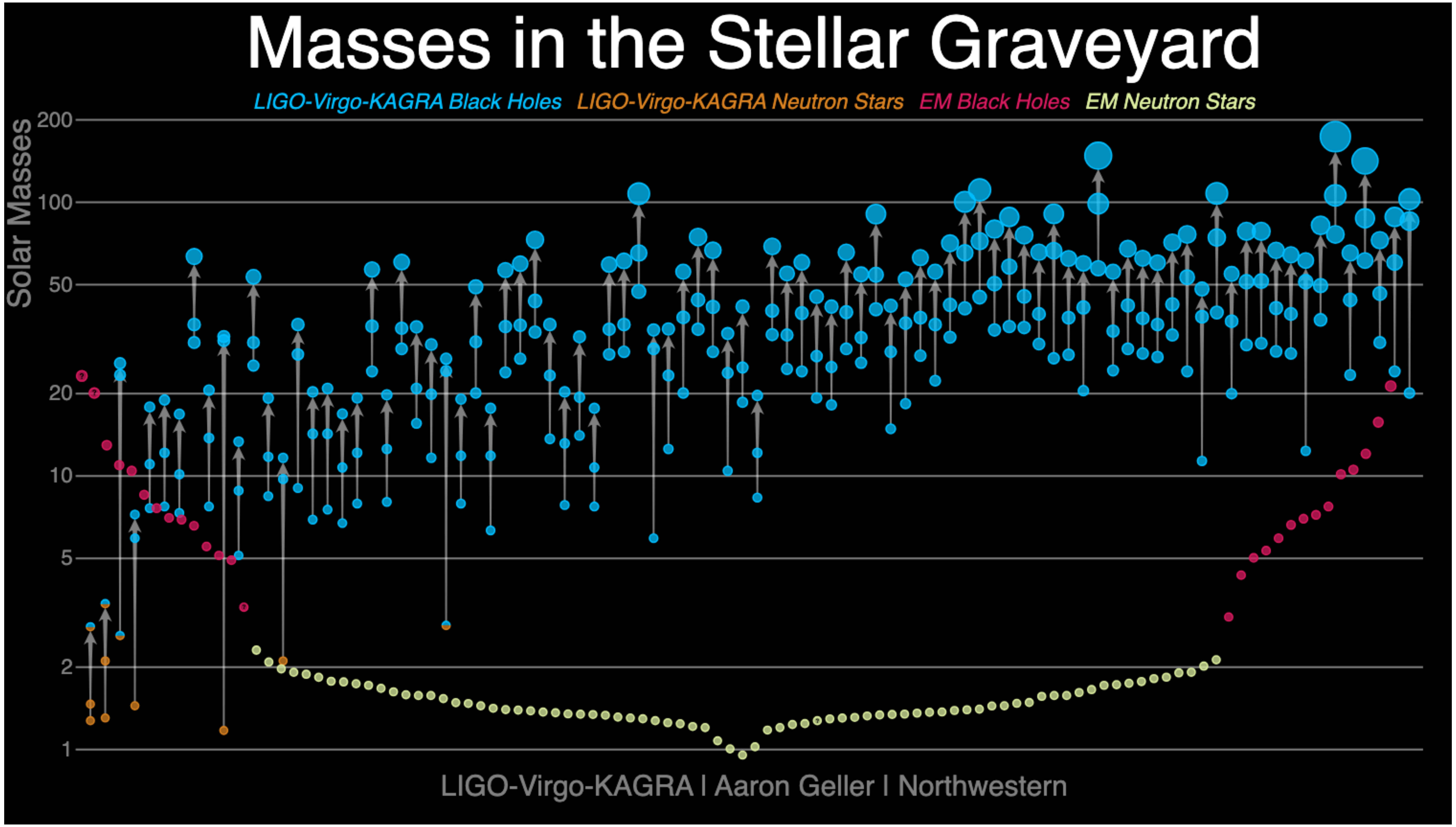
Credits: NASA

- Definitive link between BNS merger and short GRB
- Observation of a kilonova
- BNS mergers produce heavy elements
- GWs travel at the same speed as light
- Independent measure of the Hubble constant



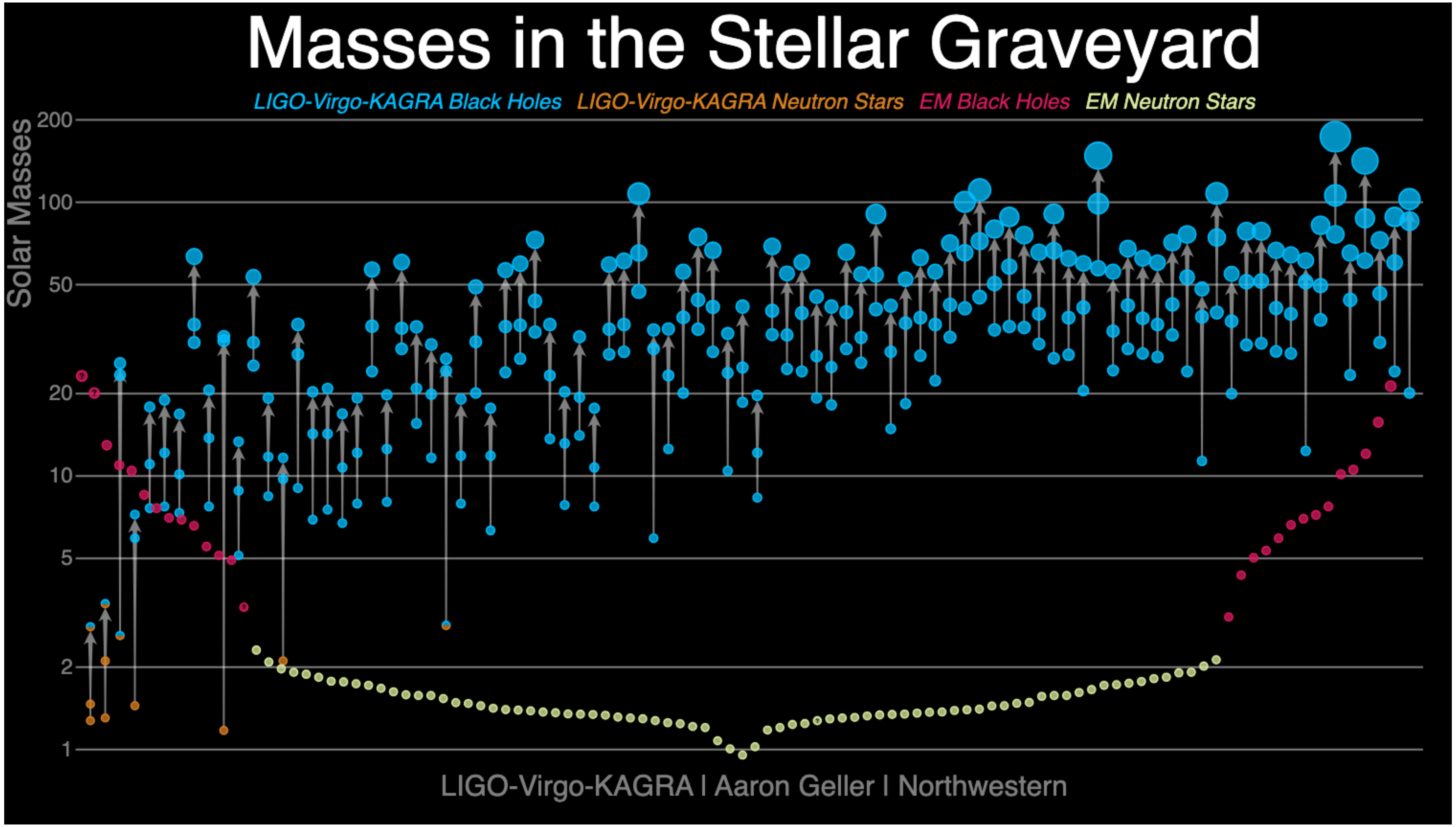
Credits: Abbott et al. 2017

GWTC-2.1 and GWTC-3



What does this chart already tell you about GW detections ?

GWTC-2.1 and GWTC-3



Population studies

Frequency

- Dynamical frequency: $f_{dyn} = \sqrt{\frac{3GM}{16\pi^2 R^3}}$
- Schwarzwild radius: $r_s = \frac{2GM}{c^2}$
- Examples:
  - **neutron star:** $M = 1.4\ \mathrm{M}_\odot$, $R = 10\ \mathrm{km}$, $f_{dyn} \sim 1.9\ \mathrm{kHz}$
  - **black hole:** $M = 10\ \mathrm{M}_\odot$, $R = 30\ \mathrm{km}$, $f_{dyn} \sim 1\ \mathrm{kHz}$
  - **super massive black holes:** $M = 4.5 \times 10^6\ \mathrm{M}_\odot$, $R = 1.3 \times 10^7\ \mathrm{km}$, $f_{dyn} \sim 2\ \mathrm{mHz}$

Refs: Schutz 1997, Camp and Cornish 2004, Ghez et al. 2008

Amplitude

- Quadrupole formula $h_{ij} = \frac{G}{c^4}\frac{2}{r}\left[\ddot{Q}_{ij}\right]^{TT}_{ret}$ gives the metric h_{ij} of the wave

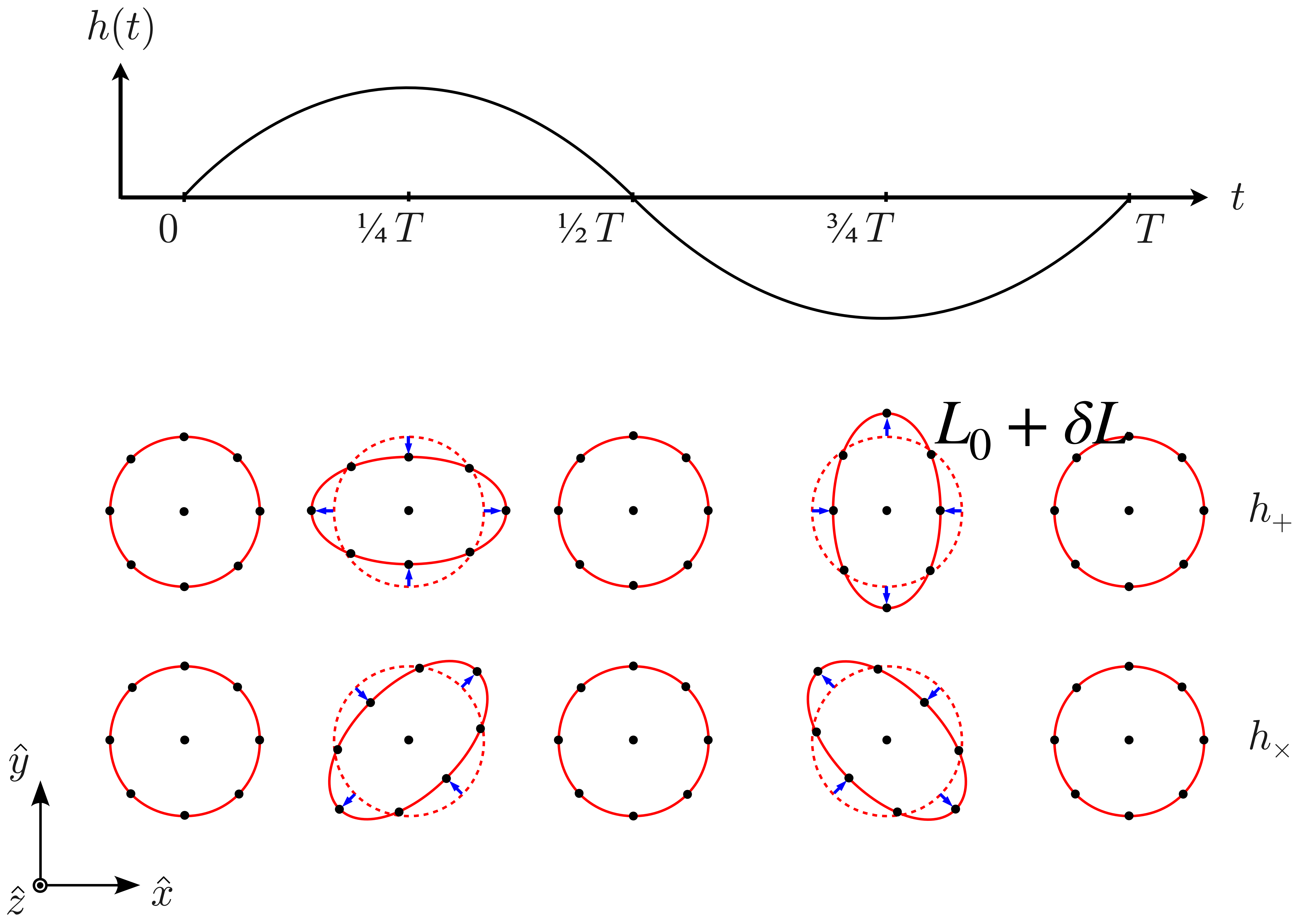
- Variation of quadrupole moment: $|\ddot{Q}_{ij}| \leq \frac{d^2}{dt^2}\int \rho x^3 dx \sim (2\pi f_{dyn})^2 MR^2$

- GW amplitude of a **binary system**: $h \leq 2\left(\frac{2GM}{Rc^2}\right)\left(\frac{GM}{rc^2}\right)$ where M is the total mass, R is the separation between the sources, r is the distance from the source

Refs: Schutz 1984, Schutz 1997

Polarisation

- Gravitational waves have two polarisations, + and ×
- Effect of polarisations on a ring of freely falling particles ➡️
- $\delta L \sim \frac{1}{2}\, h\, L_0$
- This means for a detector on Earth of $L_0 = 1\ \mathrm{km}$, $\delta L \sim 10^{-18}$ m due to GW. [Size of an atomic nucleus $\sim 10^{-15}$ m]



Credits: Le Tiec and Novak 2016

BNS GW

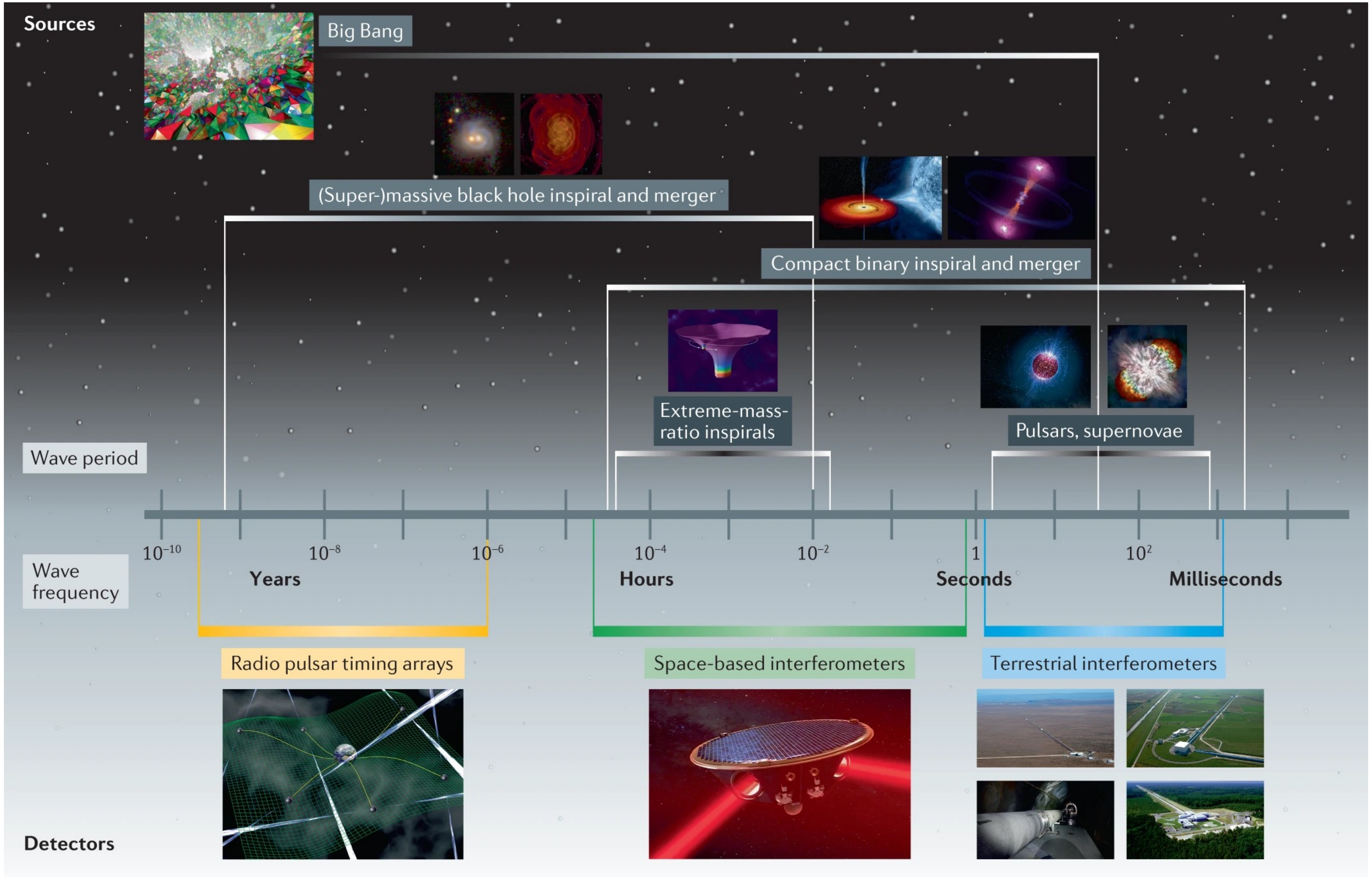
- Binary system consisting of two neutron stars of equal mass of $1.4\ \mathrm{M}_\odot$ separated by $90\ \mathrm{km}$ at a distance of $15\ \mathrm{Mpc}$

- $h \sim 10^{-21} \left(\dfrac{15\ \mathrm{Mpc}}{r}\right)\left(\dfrac{M}{2.8\ \mathrm{M}_\odot}\right)^2 \left(\dfrac{90\ \mathrm{km}}{R}\right)$

- $f = \left(\dfrac{M}{2.8\ \mathrm{M}_\odot}\right)^{1/2} \left(\dfrac{90\ \mathrm{km}}{R}\right)^{3/2}\ 100\ \mathrm{Hz}$

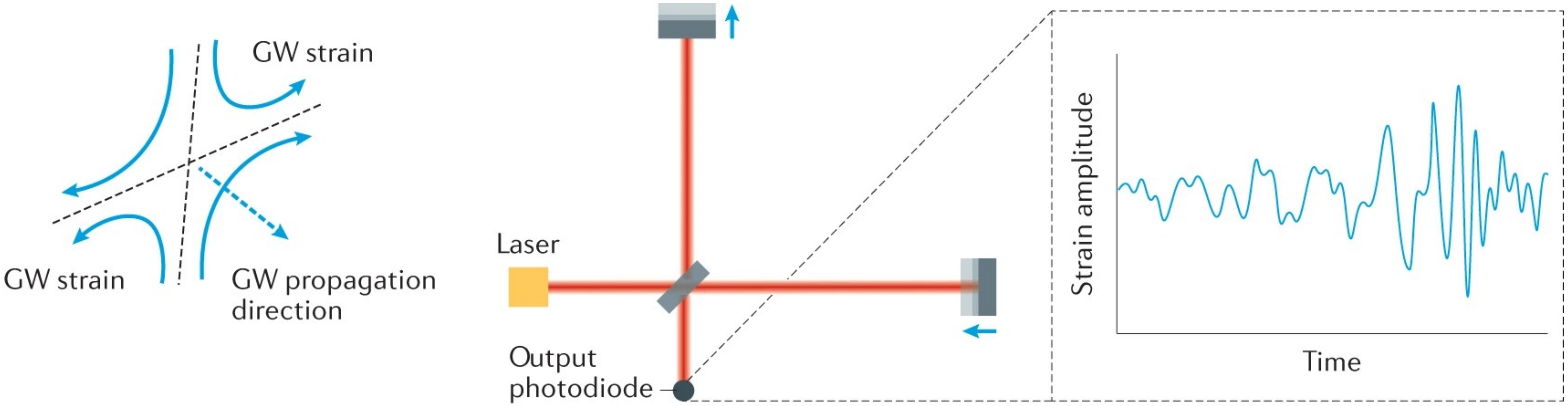
Refs: Schutz 1997

GW observation band



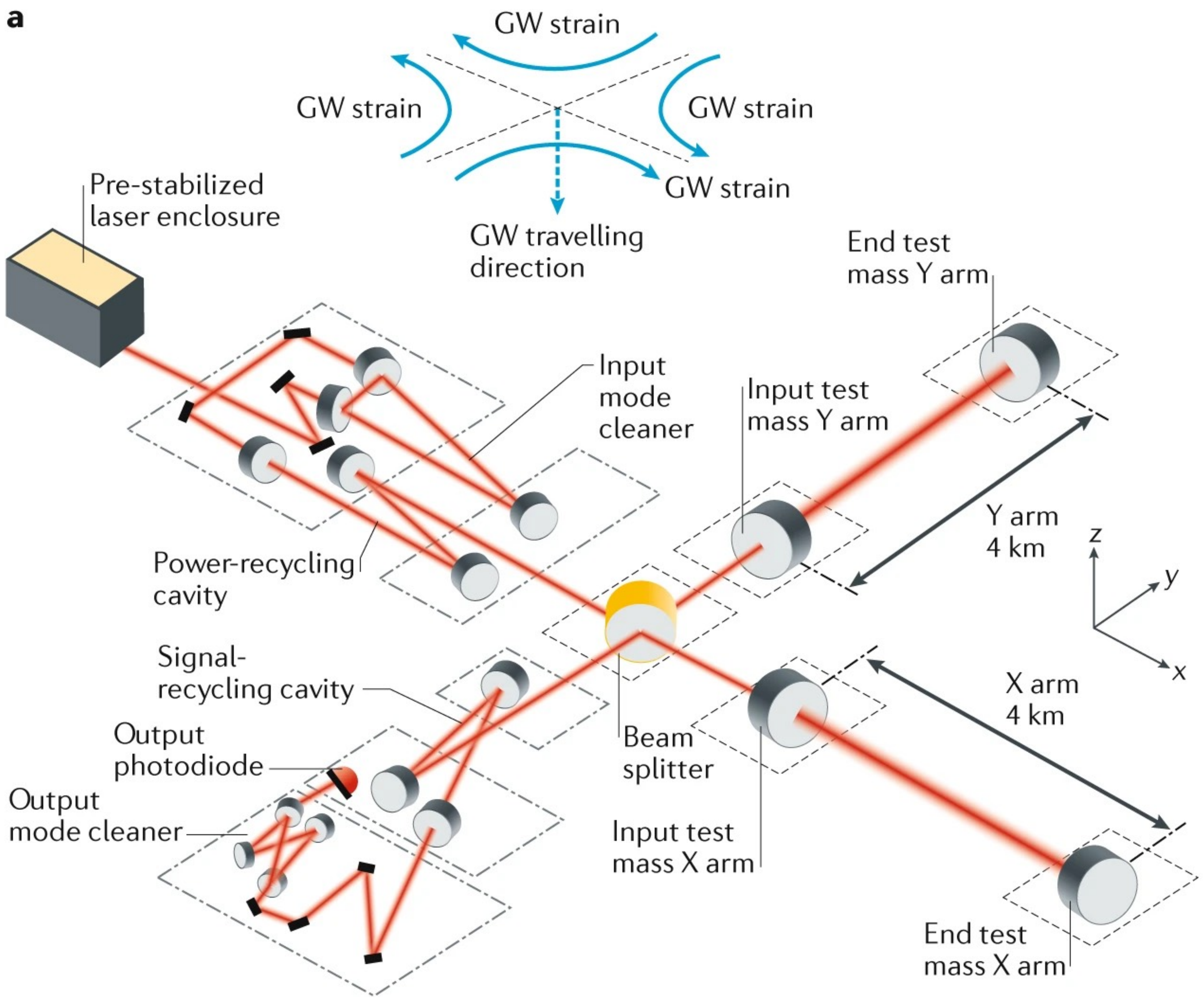
Credits: Bailes et al. 2021

GW detection



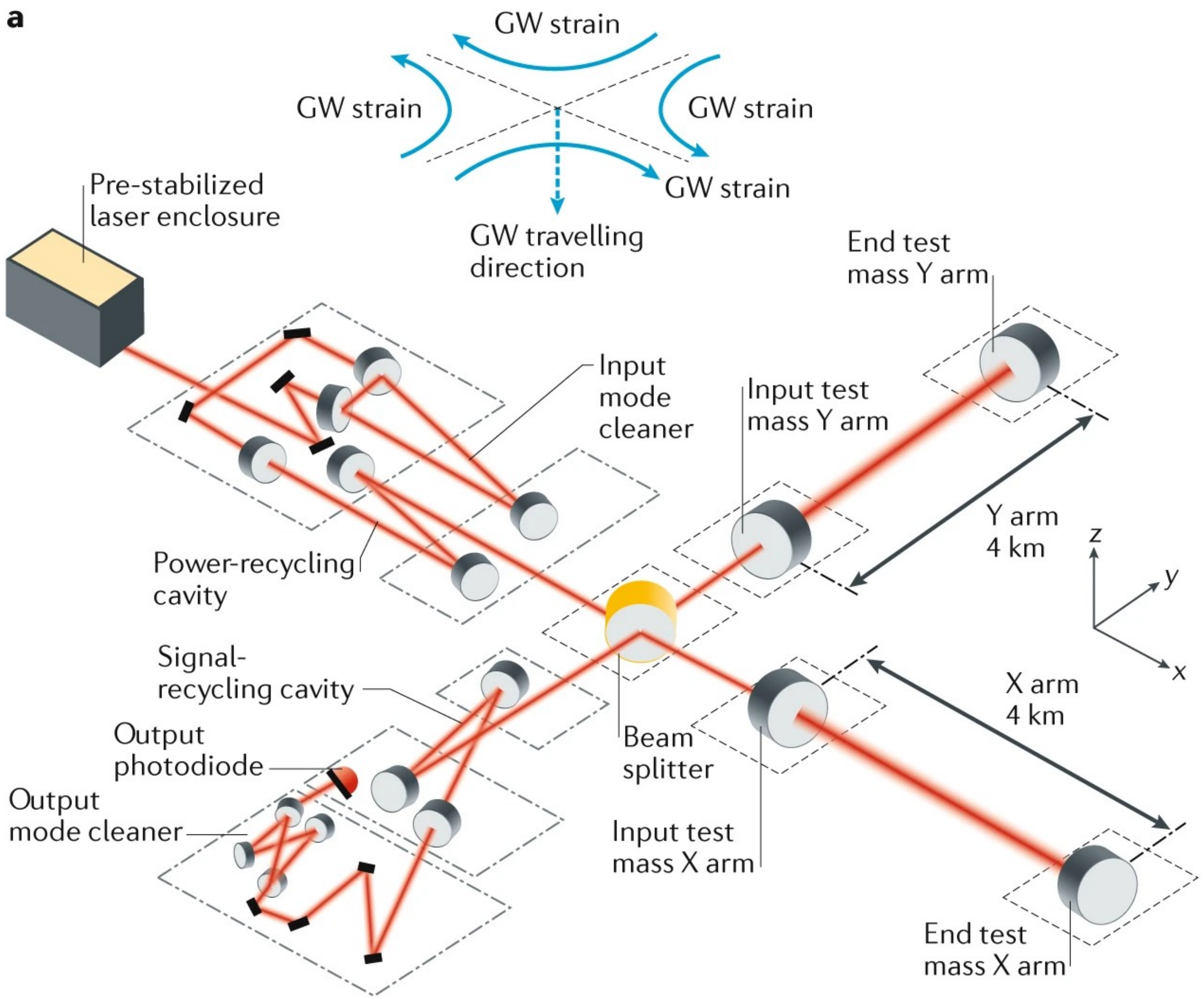
Credits: Bailes et al. 2021

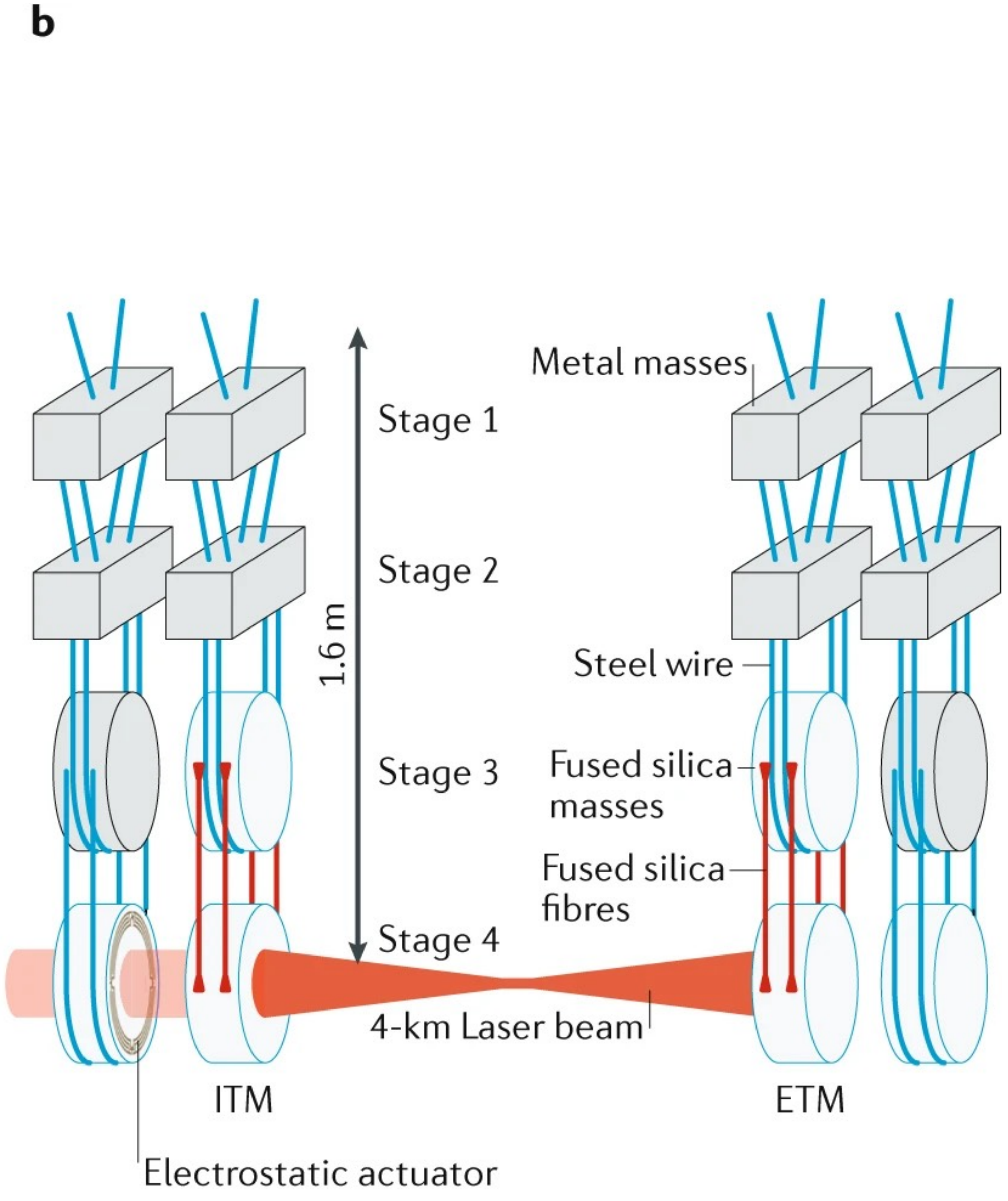
Detector: Michelson interferometer



Credits: Bailes et al. 2021

Detector : super attenuator

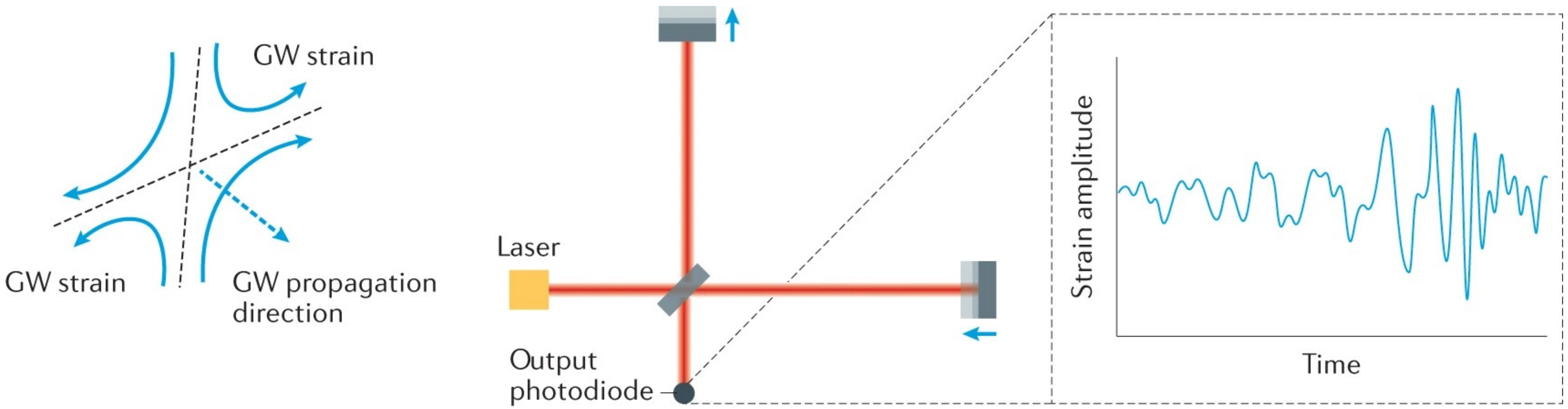


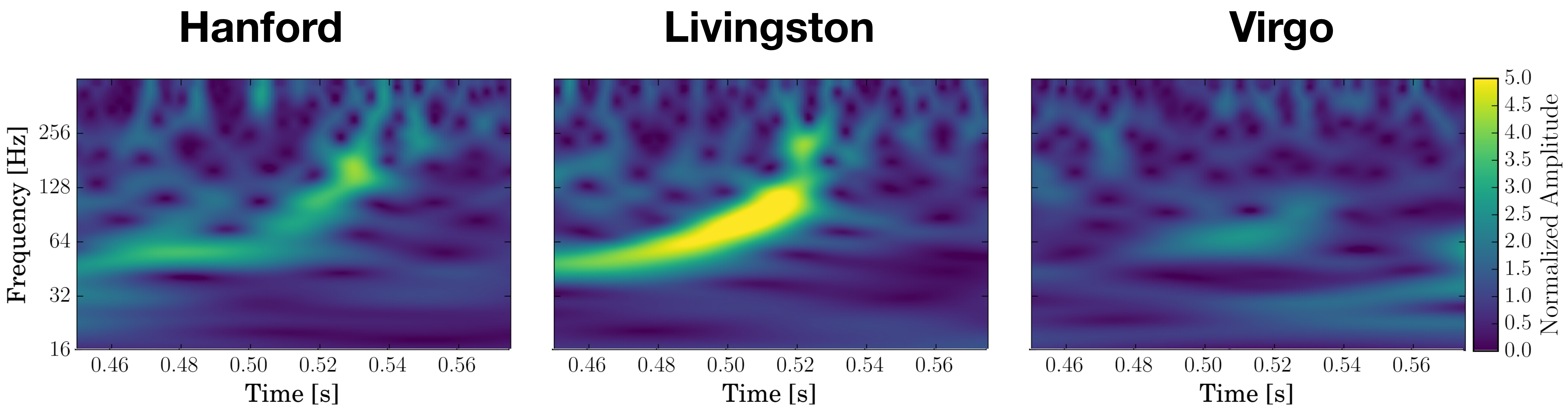


For a more interactive explanation see here (3.40 min)

Credits: Bailes et al. 2021

GW signal

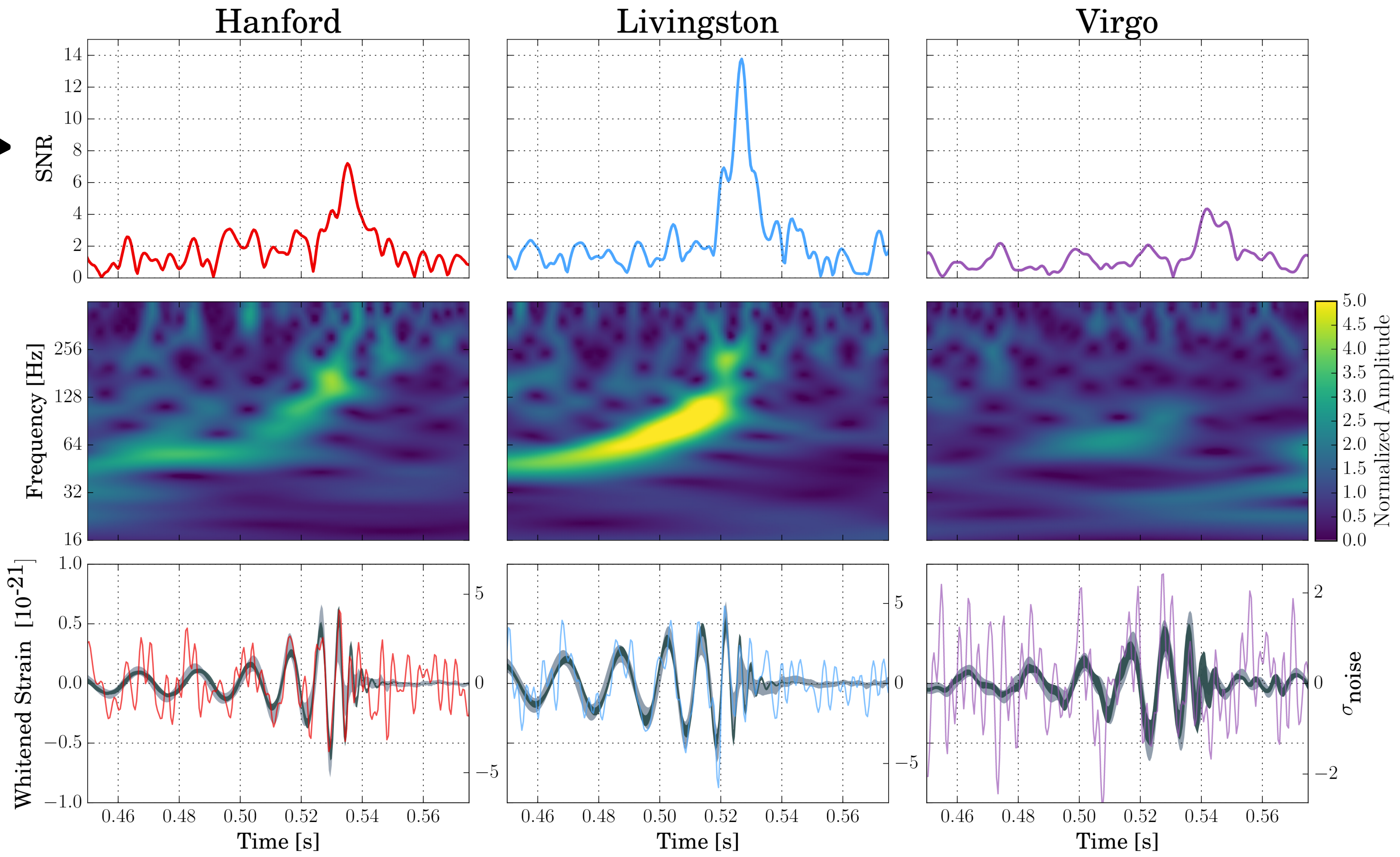




Credits: Ricci 2019

Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df \longrightarrow$$

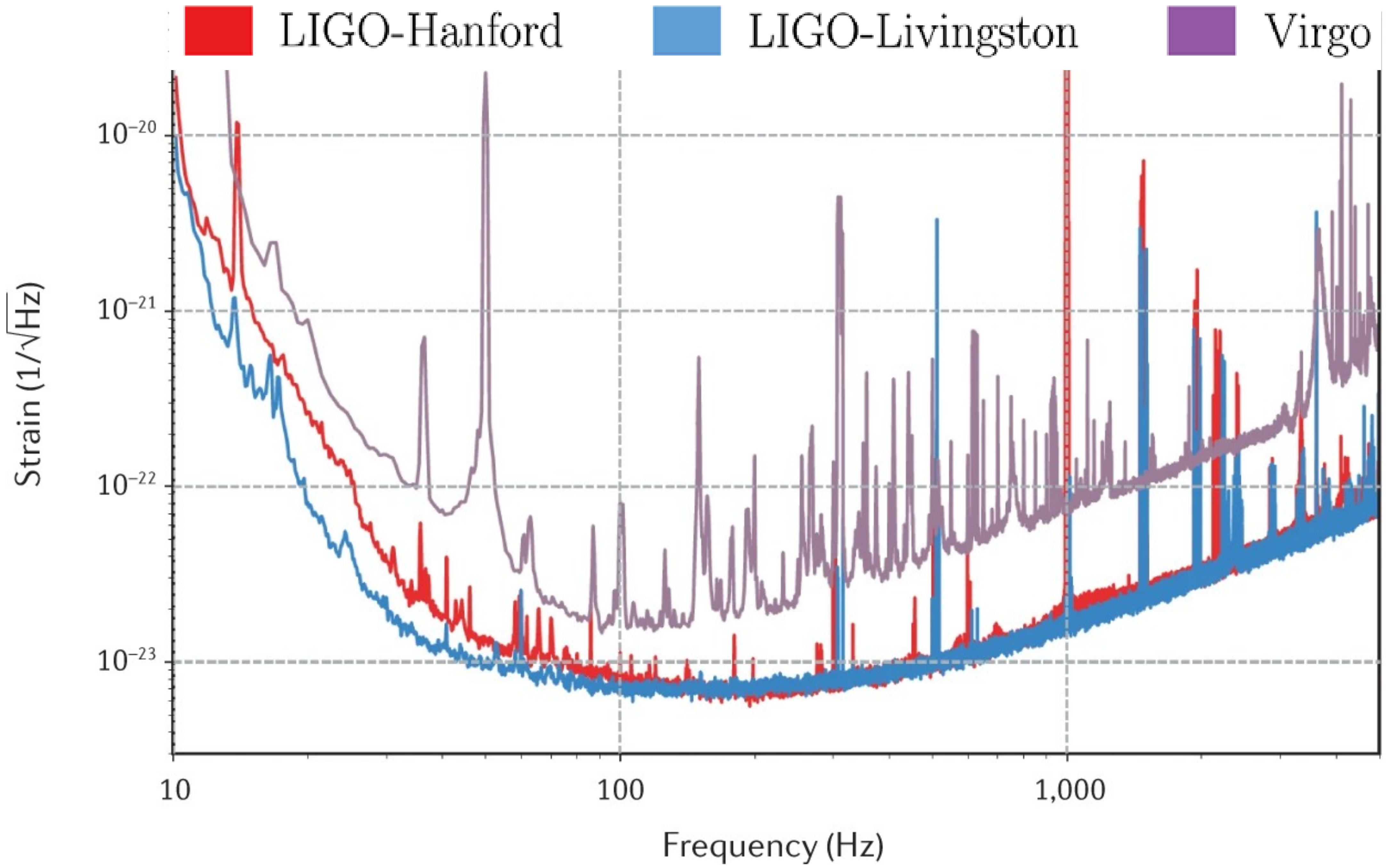


Finding the maximum of SNR

Credits: Ricci 2019

Matched filtering

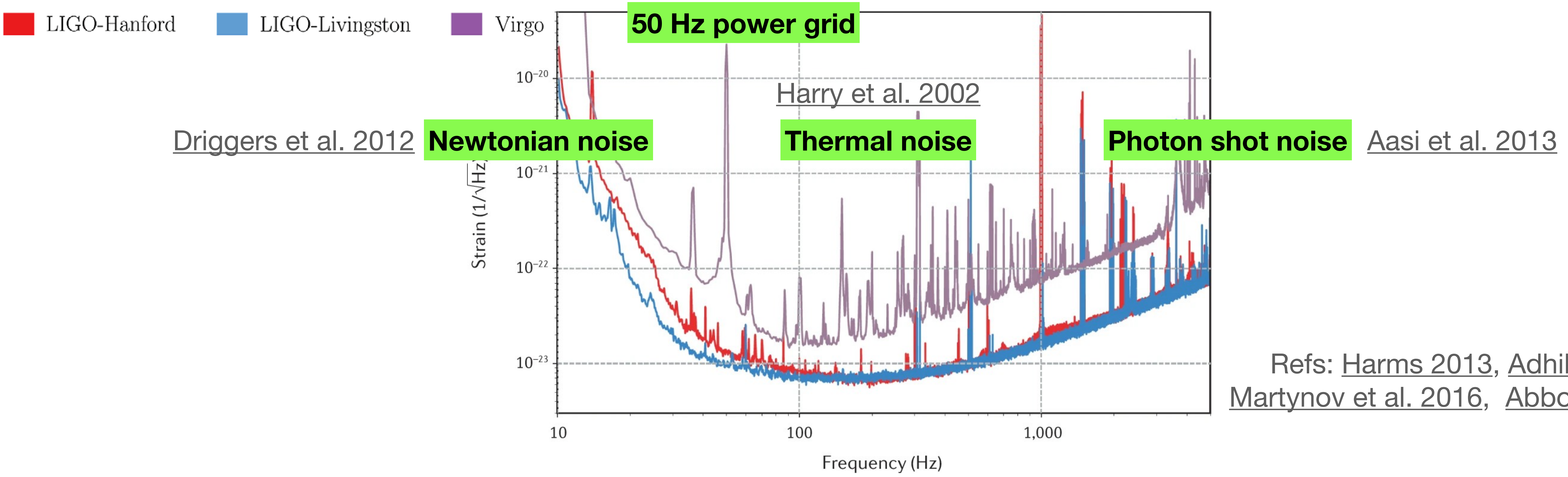
$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)}\, df$$



Credits: Bailes et al. 2021

Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)}\, df$$



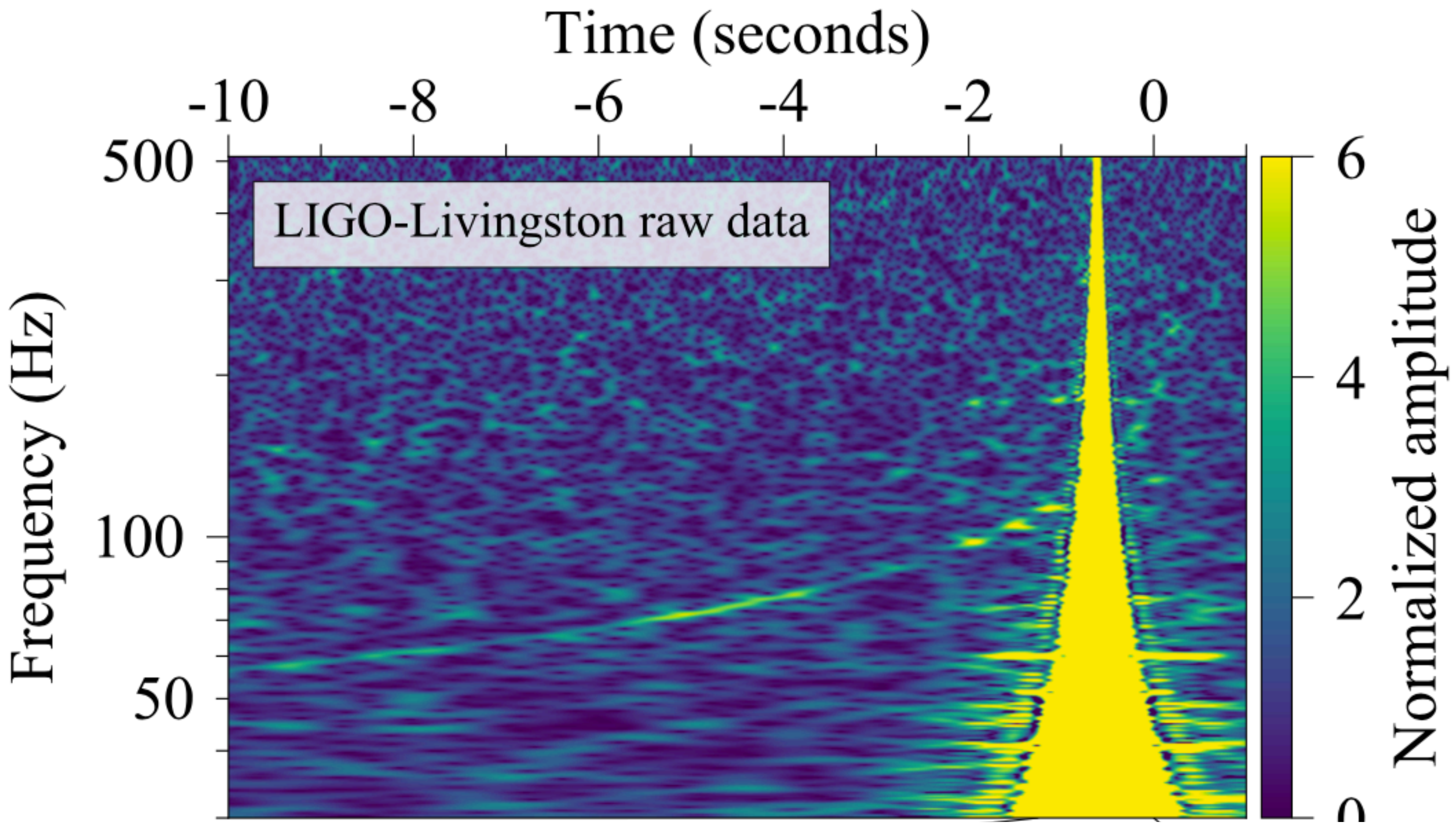
Refs: Harms 2013, Adhikari 2014, Martynov et al. 2016, Abbott et al. 2017

Credits: Bailes et al. 2021

Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)}\, df$$

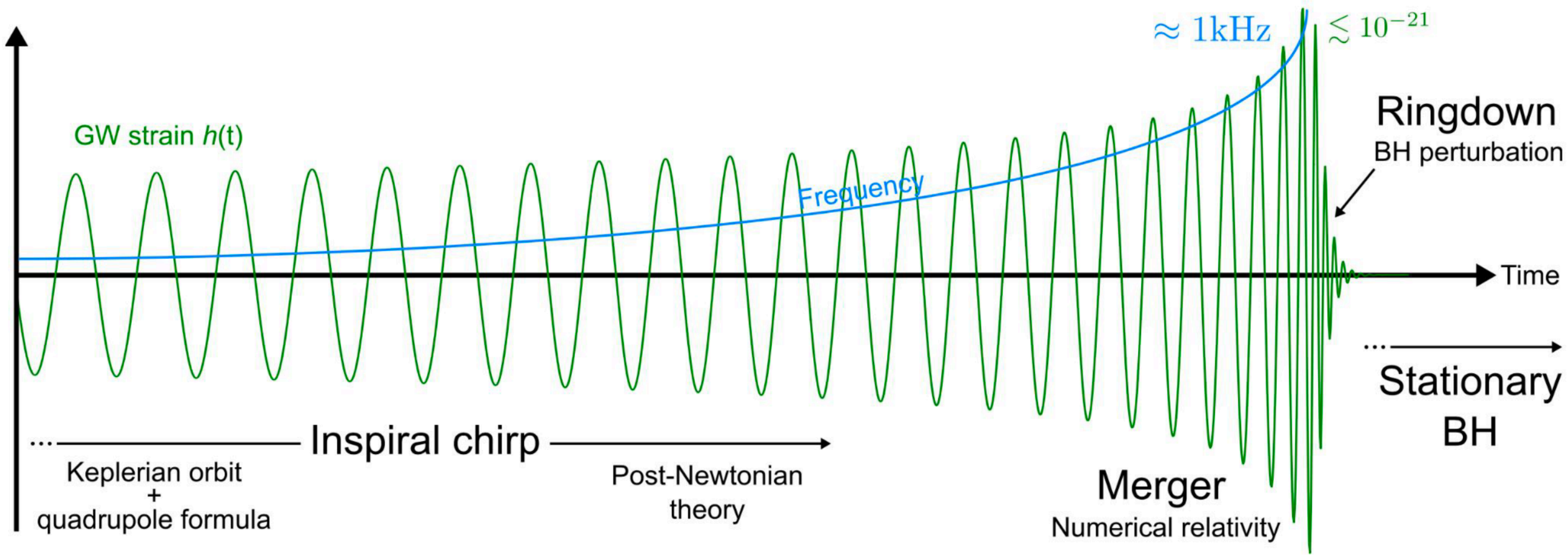
$$s(t) = n_{\mathrm{Gaussian}}(t) + n_{\mathrm{non-Gaussian}}(t) + h(t)$$



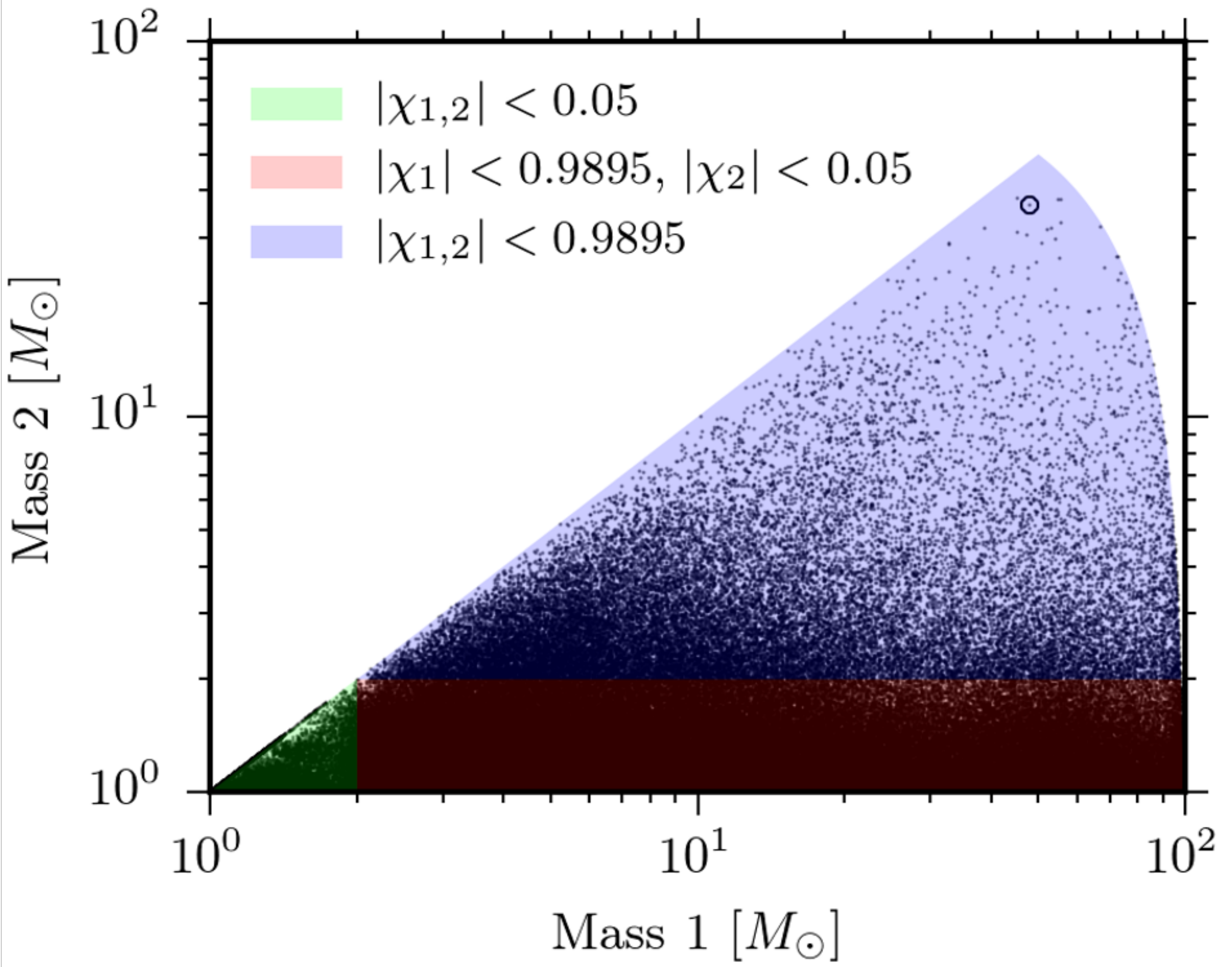
GW170817 Credits: Abbott et al. 2017

Matched filtering

$$\rho \sim \int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)}\, df$$



Credits: Tito Dal Canton

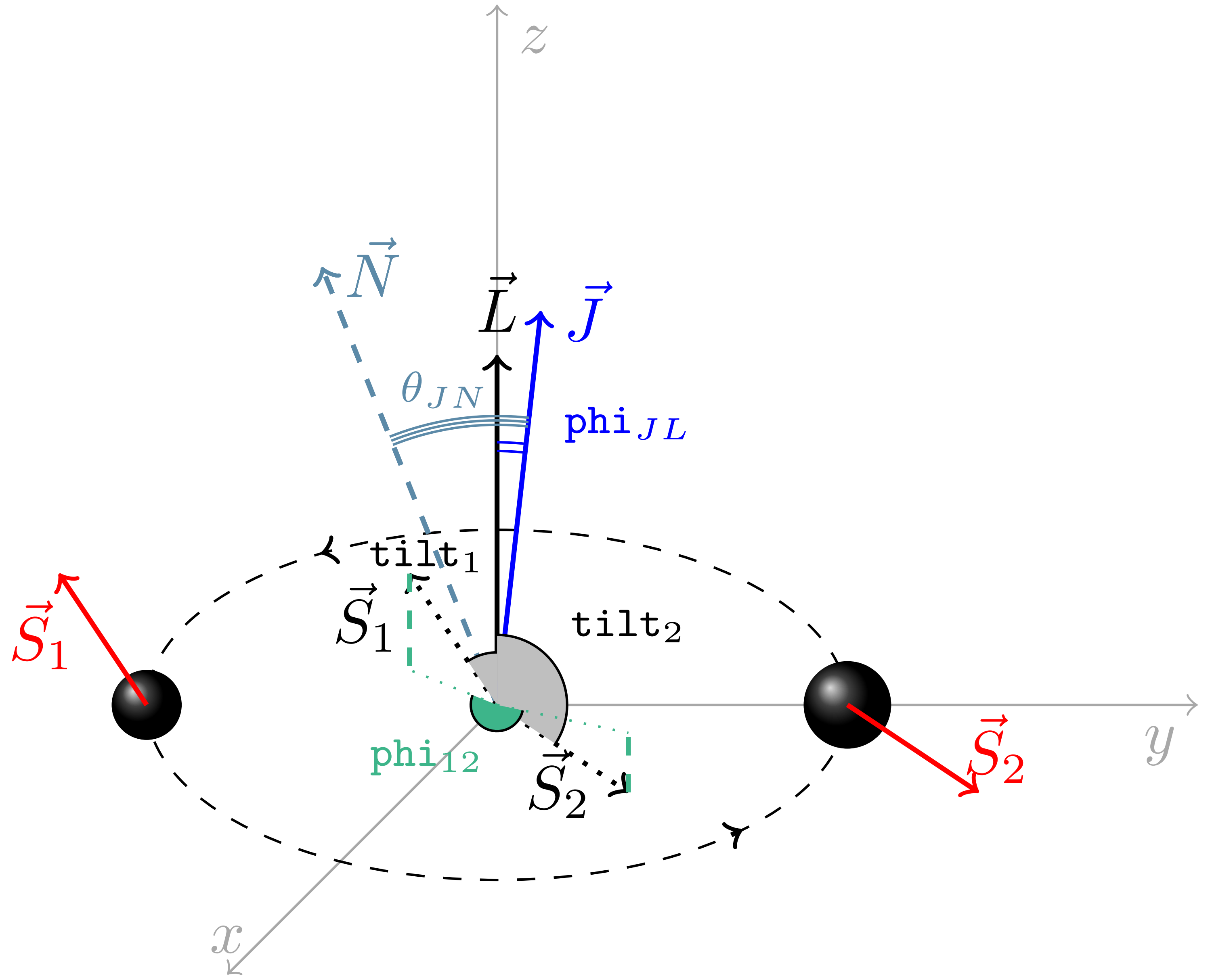


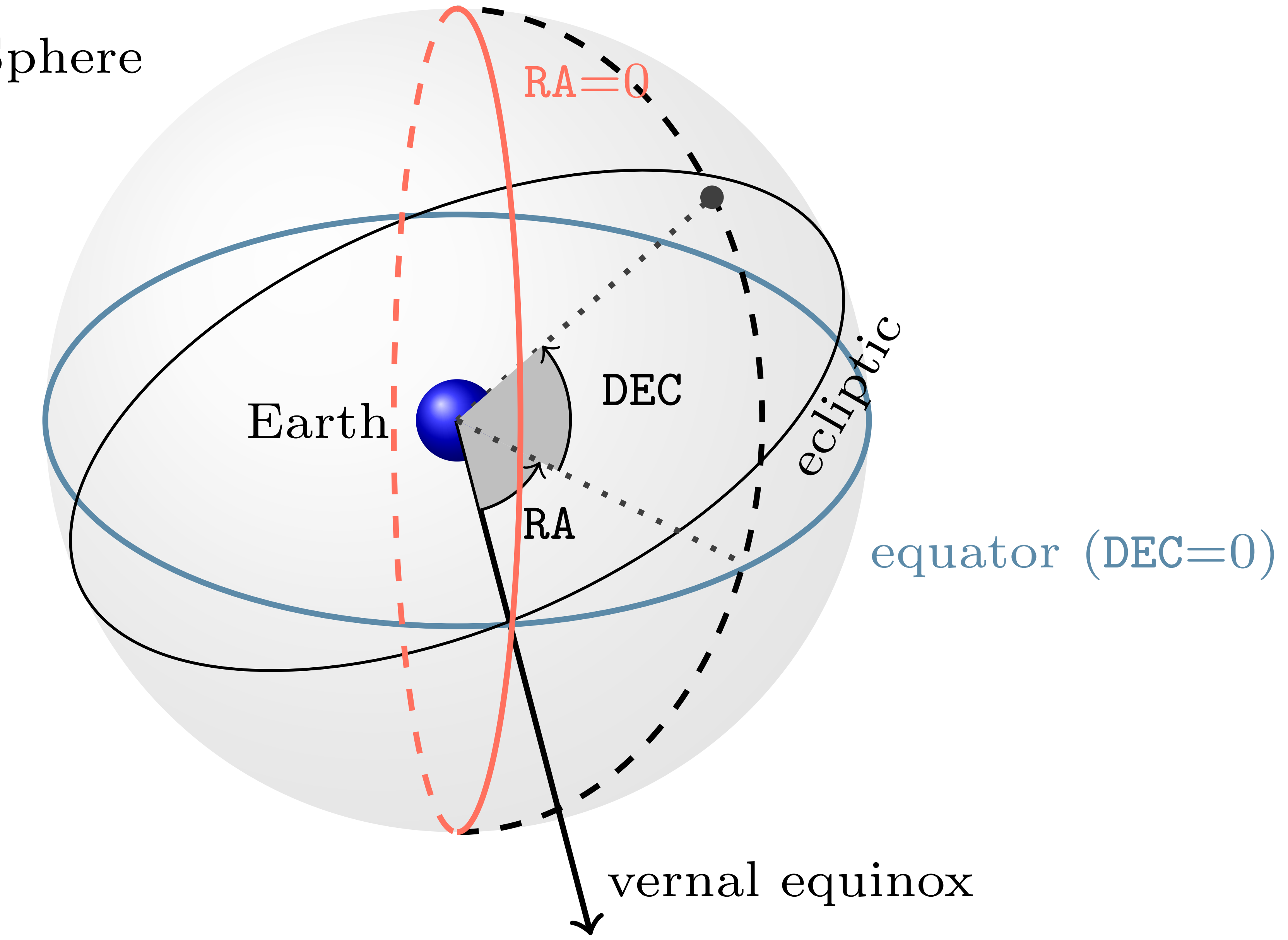
See here (30 s) why it is called chirp

Credits: LIGO-Virgo-Collaboration

Parameters

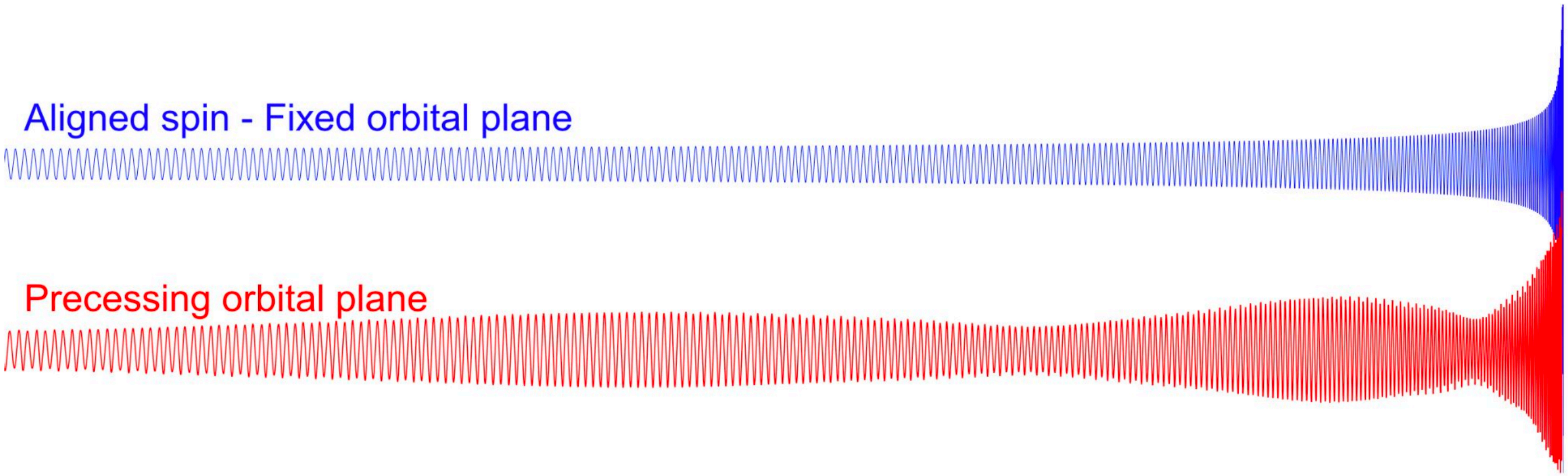
- A BBH waveform has in total 16 parameters
  - **8 intrinsic**: masses and spins
  - **8 extrinsic**: source location, inclination angle, luminosity distance, polarisation angle, orbital phase, merger time and eccentricity
  - $\theta = \{m_1, m_2, \chi_{1,x}, \chi_{1,y}, \chi_{1,z}, \chi_{2,x}, \chi_{2,y}, \chi_{2,z}, \mathrm{ra}, \mathrm{dec}, \iota, d_\mathrm{L}, \psi, \phi_c, t_c, e\}$





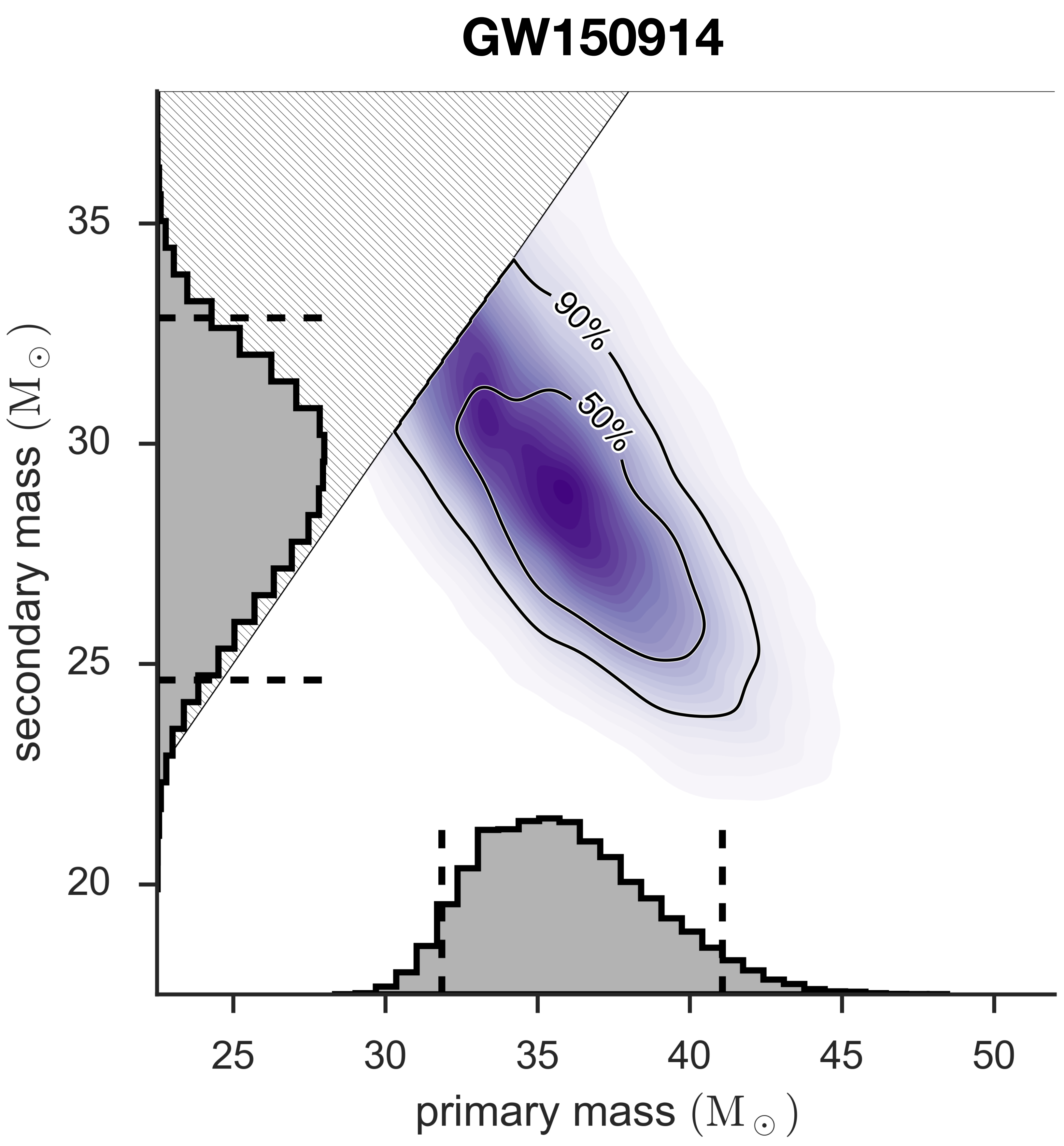
Credits: Dupletsa et al. 2024

Effect of parameters



Credits: Tito Dal Canton

Parameter estimation

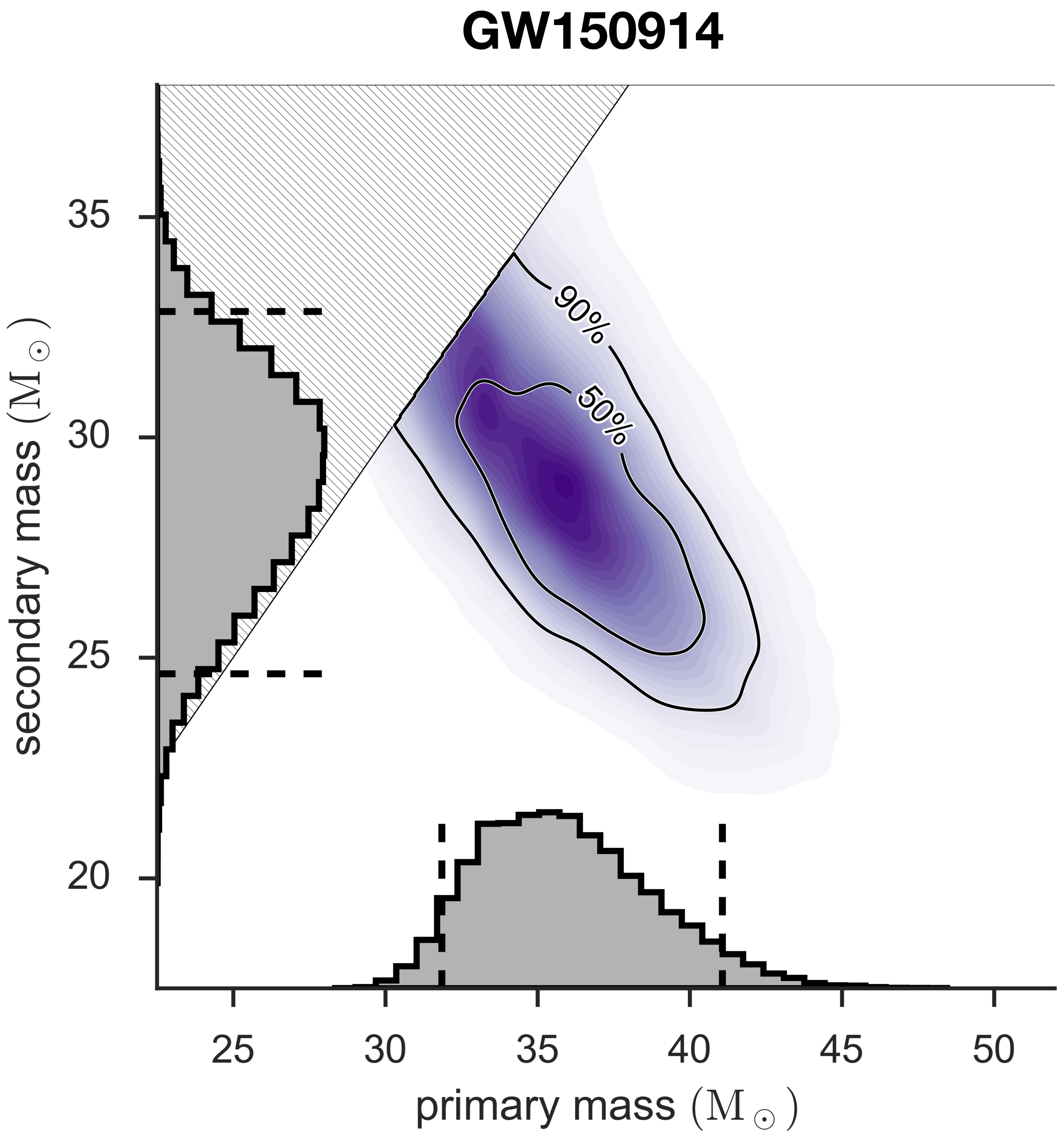


Posterior

$$\boxed{p(\theta|d)} = \frac{\mathscr{L}(d|\theta)p(\theta)}{p(d)}$$

Credits: LIGO-Virgo-KAGRA collaboration

Parameter estimation



Prior

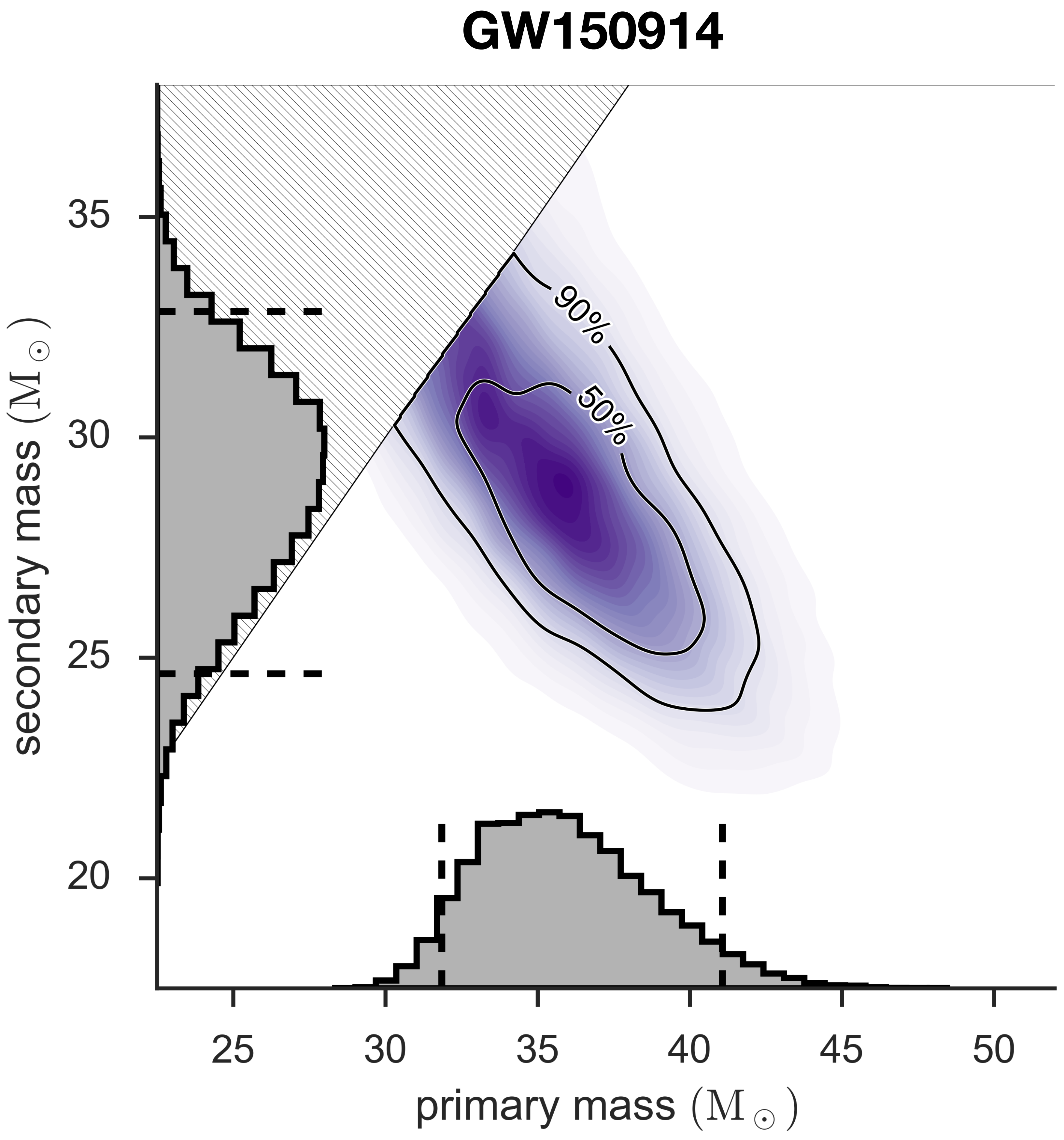
Posterior

$$\boxed{p(\theta|d)} = \frac{\mathscr{L}(d|\theta)\boxed{p(\theta)}}{p(d)}$$

What is a good choice for priors?

Credits: LIGO-Virgo-KAGRA collaboration

Parameter estimation



Likelihood
Prior
Posterior
Evidence

$$\boxed{p(\theta|d)} = \frac{\boxed{\mathscr{L}(d|\theta)}\boxed{p(\theta)}}{\boxed{p(d)}}$$

$$\mathscr{L} \propto \exp\left[-\frac{1}{2}(d - h(\theta)\,|\,d - h(\theta))\right]$$

Inner product $$(a\,|\,b) = 4\Re\int_0^\infty \frac{a(f)b^*(f)}{S_n(f)}\mathrm{d}f$$

Credits: LIGO-Virgo-KAGRA collaboration

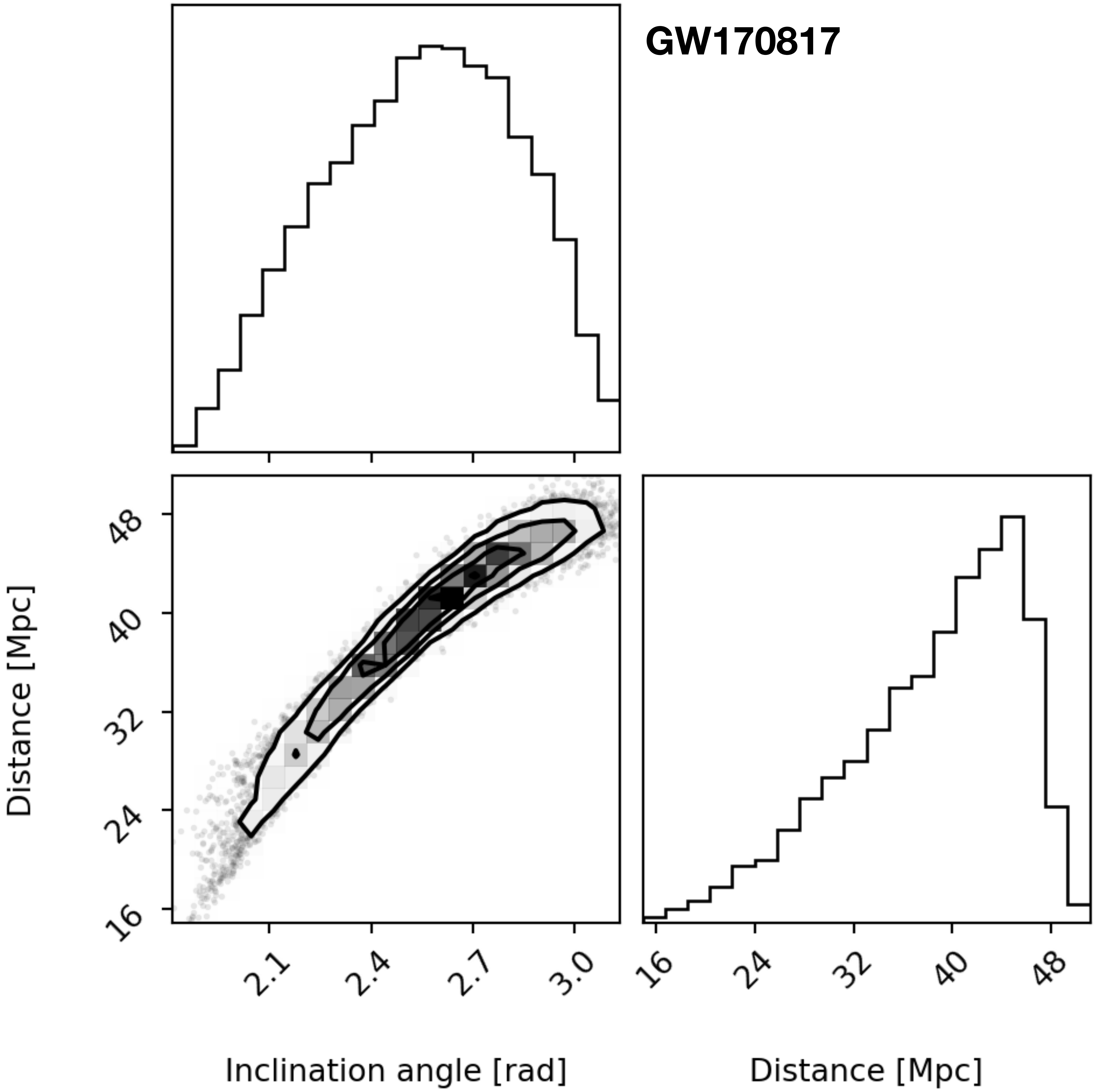
Ref: Cutler and Flanagan 1994

Parameter Estimation challenge

- $p(\theta | d)$ has no analytical solution

- Necessary to **sample** (MCMC, nested sampling)

- Once PE is done, necessary do deal with **degeneracies**



Credits: Jacopo Tissino, Ulyana Dupletsa

Fisher approximation matrix

- Introducing today's hands-on on GWFISH
- Approximation of the likelihood $\mathscr{L}(d\,|\,\theta) \propto \exp\left[-\frac{1}{2}\Delta\theta^i \mathscr{F}_{ij}\Delta\theta^j\right]$
- Where $\mathscr{F}_{ij} = (\partial_i h\,|\,\partial_j h) = 4\mathfrak{R}\int \frac{1}{S_n(f)}\frac{\partial h}{\partial \theta_i}\frac{\partial h^*}{\partial \theta_j}\mathrm{d}f$

What you did (not) learn today

- GW detection
- Match filtering
- Parameter estimation of single events

Tomorrow

- Population studies
- Astrophysical formation of GW sources

Further reading

- This lecture is based on lecture materials from Marica Branchesi, Jan Harms, Tito Dal Canton, Michela Mapelli, Giuliano Iorio and Eleonora Loffredo

- Books: Gravitational Waves: Volume 1, Gravitational Waves: Volume 2

- **See you this afternoon for the hands-on section!**